Warm-up 1: Let \( f(x) = \frac{x^2 - 6x + 8}{x - 2} \) and let \( g(x) = x - 4 \). Are \( f \) and \( g \) equivalent functions? Why or why not? Hint: Try factoring the numerator of \( f(x) \).

Draw the graphs of \( y = f(x) \) and \( y = g(x) \).
**Limits:**

There is a technical definition, but for us, the intuitive definition will do just fine. Here it is:

The limit $L$ is the value the function “gets close to” if we make the $x$ values “get close to” (but not equal to) $a$. We write $\lim_{x \to a} f(x) = L$.

**Continuity:**

**Definition:** A function $f(x)$ is continuous at $x = a$ if and only if $\lim_{x \to a} f(x) = f(a)$.

This definition actually states three things:

1.

2.

3.
Part 1: Limits and Continuity given by a Graph

Let $y = f(x)$ be given by the graph below. Use the graph to find the following.

1. a. $f(1) =$
   
   b. $\lim_{x \to 1} f(x) =$
   
   c. Is $f$ continuous at $x = 1$?
   
   d. $\lim_{x \to 1} (f(x) + 3x) =$

2. a. $\lim_{x \to 2} f(x) =$
   
   b. $\lim_{x \to -2} f(x) =$
   
   c. Is $f$ continuous at $x = -2$?

3. a. $\lim_{x \to -5} f(x) =$
   
   b. $\lim_{x \to -5} f(x) =$
   
   c. $\lim_{x \to -5} f(x) =$
   
   d. $f(-5) =$
   
   e. Is $f$ continuous at $x = -5$?
   
   f. $\lim_{x \to -5} (4f(x)) =$

4. a. $\lim_{x \to 3} f(x) =$
   
   b. $\lim_{x \to 3} f(x) =$
   
   c. $\lim_{x \to 3} f(x) =$
   
   d. $f(3) =$
   
   e. Is $f$ continuous at $x = 3$?
   
   f. $\lim_{x \to 3} \sin \left( \frac{\pi}{3} f(x) \right) =$

5. a. $\lim_{x \to 6} f(x) =$
   
   b. $\lim_{x \to 6} f(x) =$
   
   c. $\lim_{x \to 6} f(x) =$
   
   d. $f(6) =$
   
   e. Is $f$ continuous at $x = 6$?
   
   f. $\lim_{x \to 6} f(x) =$
   
   g. $\lim_{x \to 6} f(x) =$

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Part 2: Limits and Continuity given by an Equation

Hints about finding Limits:

- Plugging in nearby numbers will get you no credit unless the directions specifically say to do so.
- For any function which is CONTINUOUS, you can find the limit just by plugging in the number as long as the answer is defined.
  - In particular we know the following functions and all their combinations are continuous wherever they are defined:
    - ________________________________  ____________________________
    - ________________________________  ____________________________
    - ________________________________  ____________________________
    - ________________________________
  - Ways you can combine continuous functions to get another continuous function:
    - ________________________________  ____________________________
    - ________________________________  ____________________________
    - ________________________________  ____________________________
  - What do you do if the function is undefined? Look at the form of the limit!

<table>
<thead>
<tr>
<th>zero</th>
<th>nonzero number</th>
<th>zero</th>
</tr>
</thead>
<tbody>
<tr>
<td>nonzero number</td>
<td></td>
<td>zero</td>
</tr>
<tr>
<td>number</td>
<td>infinity</td>
<td>infinity</td>
</tr>
<tr>
<td>infinity</td>
<td></td>
<td>infinity</td>
</tr>
</tbody>
</table>

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Limits Problems:

a) \( f(x) = \frac{1}{x-1} \)

a) \( \lim_{x \to 1^-} f(x) = \)

b) \( \lim_{x \to 1^+} f(x) = \)

c) \( \lim_{x \to 1} f(x) = \)

d) \( \lim_{x \to \infty} f(x) = \)

e) List the asymptotes of \( f \). Justify your answers by citing appropriate limits. Explain how you know there are no others.

b) \( f(x) = \frac{x^2 - 1}{x + 1} \)

a) \( \lim_{x \to -1^-} f(x) = \)

b) \( \lim_{x \to -1^+} f(x) = \)

c) \( \lim_{x \to -1} f(x) = \)

d) \( \lim_{x \to \infty} f(x) = \)

c) \( h(x) = \frac{x^2 - 3x - 4}{2x^2 - 4x - 6} = \frac{(x-4)(x+1)}{2(x+1)(x-3)} \)

a) \( \lim_{x \to -1} f(x) = \)

b) \( \lim_{x \to 3} f(x) = \)
c) \( \lim_{x \to 4} f(x) = \)

d) \( \lim_{x \to \infty} f(x) = \)

e) List the asymptotes of \( f \). Justify your answers by citing appropriate limits. Explain how you know there are no others.

d) \( h(x) = \begin{cases} 
-x^2 + 9, & x \leq -2 \\
-2x + 1, & -2 < x < 2 \\
x + 1, & x \geq 2
\end{cases} \)

a) \( \lim_{x \to -2} f(x) = \)

b) \( \lim_{x \to 2} f(x) = \)

c) \( \lim_{x \to 1} f(x) = \)

e) \( \lim_{x \to 0} \frac{x}{\sqrt{x + 1} - 1} \)
# Comparison Chart of Limits vs. Continuity

<table>
<thead>
<tr>
<th></th>
<th><strong>Limits</strong></th>
<th><strong>Continuity</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conceptually</strong></td>
<td>Where is the function headed (y-value) as you get near a certain x-value?</td>
<td>Can you draw it without picking up your pencil?</td>
</tr>
<tr>
<td><strong>Graphically</strong></td>
<td>No jumps or infinite squiggles, ignore the point itself</td>
<td>No holes, breaks, or infinite squiggles</td>
</tr>
<tr>
<td><strong>Algebraically</strong></td>
<td>1) Limits from both sides have to agree</td>
<td>1) Limits from both sides have to agree</td>
</tr>
<tr>
<td></td>
<td>2) The y-value of the point has to agree with the limit</td>
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</tr>
<tr>
<td><strong>Math Notation</strong></td>
<td>1) $\lim_{x \to a} f(x) = \lim_{x \to a} f(x)$</td>
<td>1) $\lim_{x \to a} f(x) = \lim_{x \to a} f(x)$</td>
</tr>
<tr>
<td></td>
<td>*f(x) is defined on an interval on both sides of a</td>
<td>2) $f(a)$ is defined and</td>
</tr>
<tr>
<td></td>
<td>2) $f(a) = \lim_{x \to a} f(x)$</td>
<td>$f(a) = \lim_{x \to a} f(x)$</td>
</tr>
</tbody>
</table>

* And fine print

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Squeeze Theorem

Let $f(x) = \sin (x)$. Evaluate the limit:

$$\lim_{x \to \infty} \frac{f(x + 1)}{2x^2}$$
Intermediate Value Theorem

If \( f \) is a continuous function on the closed interval \([a, b]\), and \( d \) is any value between \( f(a) \) and \( f(b) \), then there is a number \( c \) in \((a, b)\) such that \( f(c) = d \).

We will use the IVT to show that the equation

\[ 2 \log(x) = \frac{1}{\pi} \]

has a solution on the interval \((1, 10)\).

a) IVT requires a single function, \( f \). What is your choice for \( f(x) \)? ___________________________

b) **On which interval** do we need to show that \( f \) is continuous? ___________________________

c) Explain why \( f \) is continuous on that interval.

d) Evaluate \( f(1) \) and \( f(10) \).

\[ f(1) = \underline{\text{____________________}} \]

\[ f(10) = \underline{\text{___________________}} \]

e) IVT requires a number, \( d \). What is your choice for \( d \)? \( d = \underline{\text{___________________}} \)

f) Fill in the blanks appropriately (according to your answers above): \( f(\quad) < d < f(\quad) \)

g) Based on your answers above, explain how to use IVT to determine that the equation has a solution in the interval \((1, 10)\).