MSLC Workshop Series

Math 1151 – Workshop

The Fundamental Theorem of Calculus and Substitution

# First Fundamental Theorem of Calculus

* calculates the signed area under the curve from to .
* Similarly, we can find the area under the curve between and (where is a variable) with the integral .
* This integral can be thought of as a function of x! Let’s call it:

(This function is called the *accumulation function* of the original function, .)

Assume and is the function below.

-1

1

2

3

4

5

6

7

8

9

10

-2

-1

1

2

3

1. Find .
2. Find .
3. Is increasing or decreasing on the interval ?

The rate of accumulation at is equal to the value of the function being accumulated at *.* This relationship is known as the **First Fundamental Theorem of Calculus**. That is,

, where is a continuous function on .

, where is a continuous function on .

Examples:

Find .

1.

2.

3.

4.

# Second Fundamental Theorem of Calculus

What if the function we are integrating doesn’t lend itself to nice geometry calculation?

* We could use Riemann Sums

*- OR -*

* We could use the Second Fundamental Theorem, which states

, where is any antiderivative of .

(This is much more efficient than using Riemann sums, assuming we can find an antiderivative of the function.)

**Examples:**

1.

2.

# Substitution – Undoing the Chain Rule

Suppose we want to find . We know how to find , so we are going to do a **CHANGE OF VARIABLE**.

But to do this, we also need to change the into a

Which gives us a new integral that we know how to solve:

# CONVERTING ALL *x*’s to *u*’s

Most importantly, this change of variable will only work if we change EVERY so that the ENTIRE integral is in terms of .

**When Substitution Fails:**

**Back Substitution:**

**Limits of Integration:**

# Practice Problems:

1.

2.

3.

4. (Hint: remember integral means area)

5.

6.

7.

8.