**MSLC Workshop Series**  
**Math 1151 – Workshop**  
**Indeterminate Forms and L’Hopital’s Rule**

**INDETERMINATE FORMS**

*What does it mean for a limit’s form to be indeterminate?*

*Which forms are indeterminate?*  
Circle the forms which are indeterminate.

<table>
<thead>
<tr>
<th>$\infty \cdot \infty$</th>
<th>$0 \cdot \infty$</th>
<th>$# \cdot \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\infty + \infty$</td>
<td>$\infty - \infty$</td>
<td>$\infty + #$</td>
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<tr>
<td>$\frac{\infty}{\infty}$</td>
<td>$\frac{0}{\infty}$</td>
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<td>$\frac{#}{0}$</td>
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<td>$1^0$</td>
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<td>$0^0$</td>
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Determining the Form Exercises:
Write the form of each of the following limits.

1. \( \lim_{x \to 0^+} (\sin(x) \cot(x)) \)  
2. \( \lim_{x \to \infty} \frac{\arctan(x)}{x} \)

3. \( \lim_{x \to \infty} \left( \left[ \frac{1}{x} + 1 \right]^x \right) \)  
4. \( \lim_{x \to \infty} (e^x - x) \)

5. \( \lim_{x \to \infty} \left( \frac{1}{x} + 1 \right)^{\frac{1}{x}} \)  
6. \( \lim_{x \to \infty} ([\ln(1 + e^{-x})]^x) \)

7. \( \lim_{x \to \infty} (e^x + x) \)  
8. \( \lim_{x \to \infty} \left( \frac{3x^2 + 5x - 2}{6x^2 + 3x + 1} \right) \)

9. \( \lim_{x \to \frac{\pi}{2}} \left( \frac{\cos(x)}{\tan(x)} \right) \)  
10. \( \lim_{x \to \infty} \left( x \ln \left( \frac{1}{x} \right) \right) \)

11. \( \lim_{x \to \infty} \left( \left[ \arccot(x) + \pi \right]^\frac{1}{x} \right) \)  
12. \( \lim_{x \to 1} \left( \frac{x^2 - 1}{5 \ln(x)} \right) \)

13. \( \lim_{x \to 3} \left( \frac{x^2 + 2}{x^2 - x - 6} \right) \)  
14. \( \lim_{x \to 1^-} (\cot(\pi x) + \sec(x)) \)
L’HÔPITAL’S RULE
L’Hôpital’s rule is a way of dealing with indeterminate forms of the type \( \frac{0}{0} \) or \( \frac{\infty}{\infty} \).

**L’Hopital’s Rule:** If you want to know \( \lim_{x \to a} \frac{f(x)}{g(x)} \),

and IF: #1. both \( \lim_{x \to a} f(x) = 0 \) and \( \lim_{x \to a} g(x) = 0 \)

- OR -

#2. both \( \lim_{x \to a} f(x) = \infty \) and \( \lim_{x \to a} g(x) = \infty \)

THEN: \( \lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} \).

Notes:
1. This is NOT the quotient rule for derivatives
2. L’Hopital’s Rule give the WRONG answer if #1 or #2 is not satisfied.

Examples:

\[
\lim_{x \to 1} \left( \frac{x^2 - 1}{5 \ln x} \right) = \\
\lim_{x \to \infty} \left( \frac{3x^2 + 5x - 2}{6x^2 + 3x + 1} \right) = 
\]
L’Hopital’s Rule Exercises:
Evaluate the following limits, using L’Hopital’s Rule if appropriate

1. \( \lim_{x \to 0} \frac{e^x - 3x - 1}{5x} \)

2. \( \lim_{x \to 0} \frac{\sin^2 x}{\cos x - 1} \)

3. \( \lim_{x \to 0} \frac{e^x}{x^2} \)

4. \( \lim_{x \to \infty} \frac{e^x}{x^x} \)

5. \( \lim_{x \to \frac{\pi}{2}} \frac{\tan(x)}{\csc(x)} = \)

FORCING A FRACTION

L’Hopital can also help us with the other indeterminate forms if we can **force a fraction** that yields $0 \div 0$ or $\infty \div \infty$.

**Examples:**

1. \[ \lim_{x \to \infty} \left( x \tan \frac{1}{x} \right) = \]
2. \[ \lim_{x \to \infty} \left( \left[ \frac{1}{x} + 1 \right]^x \right) = \]
3. \[ \lim_{x \to \infty} (x - \ln x) = \]
Forcing a Fraction Exercises:
Evaluate the following by applying L’Hopital’s Rule if appropriate.

1. \( \lim_{x \to 0^+} x \ln(3x) \)                       2. \( \lim_{x \to \infty} e^{-x\sqrt{x}} \)

3. \( \lim_{x \to \infty} \left( \left[ \cos \left( \frac{\pi}{x} \right) \right]^{x^2} \right) = \)

4. \( \lim_{x \to 3} \frac{e^x}{x^2 - 9} = \)

5. \( \lim_{x \to \infty} \left( \csc(x) - \frac{1}{x} \right) = \)

6. \( \lim_{x \to 0^-} \left( \frac{1}{x^2} + \frac{\cos(3x)}{x^4} \right) = \)