MSLC Workshop Series
Math 1151 – Workshop
Indeterminate Forms and L'Hopital's Rule

INDETERMINATE FORMS

What does it mean for a limit's form to be indeterminate?

Which forms are indeterminate?

Circle the forms which are indeterminate.

$\infty \cdot \infty$	$0 \cdot \infty$	$\# \cdot \infty$
$\infty + \infty$	$\infty - \infty$	$\infty + \#$
$\frac{\infty}{\infty}$	$\frac{0}{\infty}$	$\frac{\#}{\infty}$
$\frac{\#}{0}$	$\frac{0}{0}$	$\frac{\infty}{0}$
1^0	1^∞	∞^0
0^0	0^∞	∞^∞

Determining the Form Exercises:

Write the form of each of the following limits.

1. $\lim\limits_{x \to 0^+} (sin(x)\, cot(x))$

2. $\lim\limits_{x \to \infty} \left(\frac{arctan(x)}{x}\right)$

3. $\lim\limits_{x \to \infty} \left(\left[\frac{1}{x} + 1\right]^x\right)$

4. $\lim\limits_{x \to \infty} (e^x - \mathrm{x})$

5. $\lim\limits_{x \to \infty} \left(\frac{1}{x} + 1\right)^{\frac{1}{x}}$

6. $\lim\limits_{x \to \infty} ([ln(1 + e^{-x})]^x)$

7. $\lim\limits_{x \to \infty} (e^x + x)$

8. $\lim\limits_{x \to \infty} \left(\frac{3x^2+5x-2}{6x^2+3x+1}\right)$

9. $\lim\limits_{x \to \frac{\pi}{2}^-} \left(\frac{cos(x)}{tan(x)}\right)$

10. $\lim\limits_{x \to \infty} \left(\mathrm{x}\, ln\left(\frac{1}{x}\right)\right)$

11. $\lim\limits_{x \to \infty} \left([\operatorname{arccot}(x) + \pi]^{\frac{1}{x}}\right)$

12. $\lim\limits_{x \to 1} \left(\frac{x^2-1}{5\, ln(x)}\right)$

13. $\lim\limits_{x \to 3} \left(\frac{x^2+2}{x^2-x-6}\right)$

14. $\lim\limits_{x \to 1^-} (cot(\pi x) + sec(x))$

L'HOPITAL'S RULE

L'Hopital's rule is a way of dealing with indeterminate forms of the type $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

L'Hopital's Rule: If you want to know $\lim\limits_{x\to a}\frac{f(x)}{g(x)}$,

and IF: #1. <u>both</u> $\lim\limits_{x\to a} f(x) = 0$ <u>and</u> $\lim\limits_{x\to a} g(x) = 0$

- OR -

#2. <u>both</u> $\lim\limits_{x \to a} f(x) = \infty$ <u>and</u> $\lim\limits_{x \to a} g(x) = \infty$

THEN: $\lim\limits_{x\to a}\frac{f(x)}{g(x)} = \lim\limits_{x\to a}\frac{f'(x)}{g'(x)}$.

Notes:

1. **This is NOT the quotient rule for derivatives**
2. **L'Hopital's Rule give the WRONG answer if #1 or #2 is not satisfied.**

<u>Examples:</u>

$$\lim_{x\to 1}\left(\frac{x^2-1}{5\ln x}\right) =$$

$$\lim_{x\to \infty}\left(\frac{3x^2+5x-2}{6x^2+3x+1}\right) =$$

L'Hopital's Rule Exercises:

Evaluate the following limits, using L'Hopital's Rule ***if appropriate***

1. $\lim\limits_{x\to 0} \frac{e^x-3x-1}{5x}$

2. $\lim\limits_{x\to 0} \frac{\sin^2 x}{\cos x-1}$

3. $\lim\limits_{x\to 0} \frac{e^x}{x^2}$

4. $\lim\limits_{x\to \infty} \frac{e^x}{x^2}$

5. $\lim\limits_{x\to \frac{\pi}{2}} \frac{\tan(x)}{\csc(x)} =$

FORCING A FRACTION

L'Hopital can also help us with the other indeterminate forms if we can **force a fraction** that yields $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

Examples:

1. $\lim_{x\to\infty}\left(x\, tan\, \frac{1}{x}\right) =$

2. $\lim_{x\to\infty}\left(\left[\frac{1}{x}+1\right]^{x}\right) =$

3. $\lim_{x\to\infty}(x - ln\, x) =$

Forcing a Fraction Exercises:

Evaluate the following by applying L'Hopital's Rule ***if appropriate***.

1. $\lim\limits_{x \to 0^+} \mathrm{x}\, ln(3\mathrm{x})$

2. $\lim\limits_{x \to \infty} e^{-x\sqrt{x}}$

3. $\lim\limits_{x \to \infty} \left(\left[cos\left(\frac{2}{x}\right) \right]^{x^2} \right) =$

4. $\lim\limits_{x \to 3} \frac{e^x}{x^2 - 9} =$

5. $\lim\limits_{x \to \infty} \left(csc(\mathrm{x}) - \frac{1}{x} \right) =$

6. $\lim\limits_{x \to 0^-} \left(\frac{1}{x^2} + \frac{cos(3x)}{x^3} \right) =$