MSLC Workshop Series
Math 1151 - Workshop
Related Rates

Example:

James places a 13 foot ladder against the wall of a house. In his haste, he climbs up without waiting for Emily to steady the ladder. The top of the ladder slides down the wall at a rate of 1 ft/s. How fast is the base of the ladder moving when it hits Emily, who is standing 12 feet away?

Ans: 5/12 ft/s

To summarize:

1. We translated the problem into math. How did we do this?
    a.
    b.
2. Once we understood the situation, we reminded ourselves of our goal. How did we organize our thinking about the goal?
    a.
    b.
3. How did we relate the information we were given to our goal?
    a.
4. How did we get ourselves closer to our goal?
    a.
5. Now what?
    a.

You try:

A cube's volume is increasing at a rate of 20 cubic feet per minute. How fast is the side length of the cube increasing at the instant when

a. the side length is 1 foot?
b. the side length is 2 feet?

Ans a: 20/3 ft/min
Ans b: 5/3 ft/min

Compare this problem to the previous one. What was the same? What was different?

Geometry of Related Rates Problems:

Here are three common geometric situations encountered in related rates problems, but there are many more!

- Pythagorean Theorem (example: the ladder problem)
- Volume or area (example: cube problem)
- Rotation Problems (example: beacon problem)

Rotation example:

A ship sails along a straight path parallel to the shore at a speed of 6 ft/s. A beacon is located on the shore 30 feet from the nearest point on the path and is kept focused on the ship as she sails. At what rate is the beacon rotating when the ship is 40 ft from the point on the path closest to the beacon?

Ans: .072 rad/s

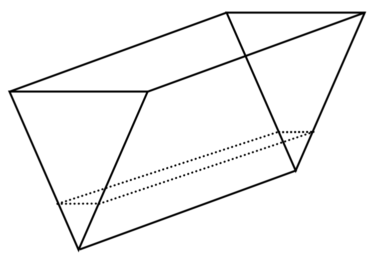
What if your equation has extra variables?

There are multiple ways to deal with this. Here are some strategies to try:

- Realize the variable is constant throughout the whole problem – plug in the number.
- Relate variables to others using geometry such as similar triangles
- Figure out what the derivative of that variable should be so that you can leave it in the problem.

Extra Variable Example:

A trough is 12 feet long and 4 feet across the top. Its ends are isosceles triangles with an altitude of 3 feet. Water is being pumped into the trough at 3 cubic feet per minute. How fast is the water level rising when it is 2 feet deep?



Ans: 3/32 ft/min

Practice Problems:

1. A child is flying a kite. If the kite is 90 feet above the child's hand level and the wind is blowing it on a horizontal course at 5 feet per second, how fast is the child letting out the string when 150 feet of string is out?

Ans: 4 ft/s

2. A lighthouse is located on an island 5 miles from the nearest point P on a straight shoreline and its light makes 6 revolutions per minute. How fast is the beam of light moving along the shoreline when it is 2 miles from P?
(Hint: 1 revolution = 2π radians)

Ans: 218.65 miles/min

3. A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of 60 cm/s. Find the rate at which the area within the circle is increasing after
   a. 1 second
   b. 3 seconds
   c. 5 seconds

Ans:_ ______
a) 7200π cm^2/s .
b) 21600π cm^2/s
c) 36000π cm^2/s

4. A boat is pulled into a dock by a rope attached to the bow (front tip) of the boat and passing through a pulley on the dock. The pulley is 3 feet higher than the bow of the boat. If the rope is pulled in at a rate of 2 ft/s, how fast is the boat approaching the dock when it is 24 feet from the dock?

Ans: $-\frac{\sqrt{585}}{12}$ft/s