MSLC Computing Derivatives Handout

Definition of the Derivative:

The derivative of a function f is another function f' whose value at any number a is: $f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$, provided that this limit exists.

Other Forms of the Definition of the Derivative:

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$	$f'(x) = \lim_{t \to x} \frac{f(t)-f(x)}{t-x}$	$f'(a) = \lim_{t \to a} \frac{f(t)-f(a)}{t-a}$

Table of Key Derivatives:

$\frac{d}{dx}\sin(x) = \cos(x)$	$\frac{d}{dx}\cos(x) = -\sin(x)$
$\frac{d}{dx}\tan(x) = \sec^2(x)$	$\frac{d}{dx}\cot(x) = -\csc^2(x)$
$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$	$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$
$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$	$\frac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2}$
$\frac{d}{dx}\sec^{-1}(x) = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\frac{d}{dx}\csc^{-1}(x) = -\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
$\frac{d}{dx}e^x = e^x$	$\frac{d}{dx}a^x = \ln(a)\,e^x$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$	$\frac{d}{dx}\log_b(x) = \frac{1}{\ln(b)\,x}$

Derivative Rules:

derivative of ANY constant (anything without an x)	$\frac{d}{dx}c = 0$
derivative of a constant times a function	$\frac{d}{dx}(c \cdot f) = c \cdot f'$
the Power Rule	$\frac{d}{dx}x^n = nx^{n-1}$
sum or difference of functions	$(f \pm g)' = f' \pm g'$
the Product Rule	$(f \cdot g)' = f' \cdot g + f \cdot g'$
the Quotient Rule	$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$
the Chain Rule	$[f(g(x)]' = f'(g(x)) \cdot g'(x)$

Implicit Differentiation

If we want to find $\frac{dy}{dx}$, we think of y as implicitly defined as a function of x.

- When we differentiate x, we get 1.
- When we differentiate y, we get $\frac{dy}{dx}$ or y' (either is fine).
- Then we solve for $\frac{dy}{dx}$.

Logarithmic Differentiation

Used when the function is complicated or for functions with an x in the base and in the exponent.

Option 1: Take the log of both sides, simplify with log properties, differentiate (implicit chain rule on y will always happen on the left side), then solve for y'.

$$y = x^x$$
$$\ln y = \ln x^x = x \ln x$$
$$\frac{\mathrm{d}}{\mathrm{dx}}(\ln \mathrm{y}) = \frac{\mathrm{d}}{\mathrm{dx}}(\mathrm{x} \ln \mathrm{x})$$
$$\frac{1}{y} y' = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$$
$$y' = y(\ln x + 1) = x^x(\ln x + 1)$$

Option 2: Rewrite your expression using $e^{\ln(expression)}$, simplify with log properties, then differentiate (not implicit).

$$y = x^x$$
$$y = e^{\ln(x^x)}$$
$$y = e^{x \ln(x)}$$
$$\frac{dy}{dx} = e^{x \ln(x)} \cdot (x \ln(x))'$$
$$\frac{dy}{dx} = e^{x \ln(x)} \cdot \left(1 \cdot \ln(x) + x \cdot \frac{1}{x}\right)$$
$$\frac{dy}{dx} = e^{\ln(x^x)} \cdot (\ln(x) + 1)$$
$$\frac{dy}{dx} = x^x(\ln(x) + 1)$$

Notice that both options yield the same result!