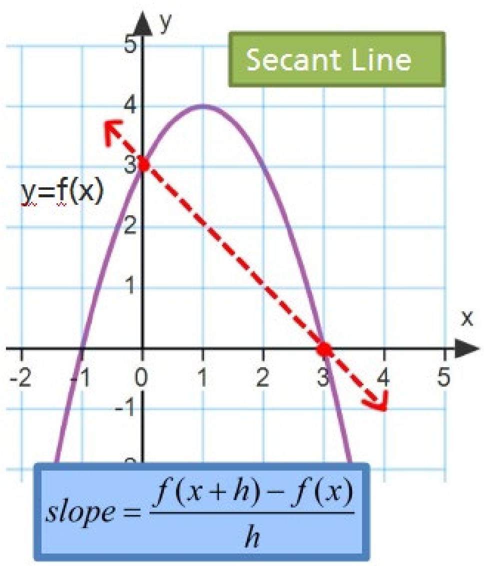
MSLC Workshop Series
Math 1151 Online Workshop
Derivatives

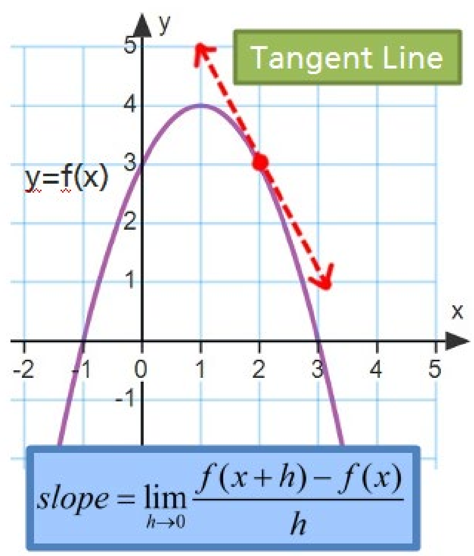
Warm-up: The slope of the line through the points (x_1, y_1) and (x_2, y_2) is given by:

The slope of the line through the points $(x, f(x))$ and $(x+h, f(x+h))$ is given by:

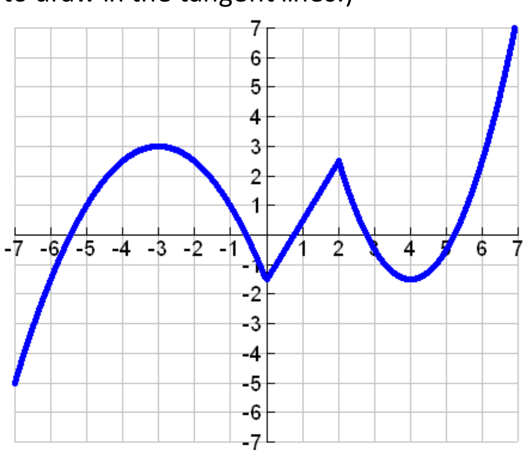
Definition of the Derivative:

We can use secant lines (over smaller and smaller intervals) to approximate the tangent line. The limit of the slope of the secant line is the slope of the tangent line. This limit is called the derivative, and a function is called *differentiable* at a point *a* if this limit exists at *a*.





Example: Use the given graph to *estimate* the value of each derivative. (Hint: think slope of tangent line. It may help to draw in the tangent lines.)



a. $f'(-3)$=
b. $f'(-1)$=
c. $f'(0)$=
d. $f'(1)$=
e. $f'(2)$=
f. $f'(4)$=

Using the Definition of the Derivative:

Hints about finding Derivatives:

- A derivative is really a limit, so start by plugging in 0 for h. If you have set it up right, you will get $\frac{0}{0}$
- There are really just three algebra tricks:
    - For polynomials: expand and cancel
    - For rational functions: find a common denominator, make into a single fraction, then treat as a polynomial
    - For roots: multiply top and bottom by the conjugate and then treat as a polynomial

Derivative Problems:

1. Let $f(x) = 2x^2 + 3x - 1$. Find $f'(x)$

2. Let $f(x) = \frac{3x}{2x+1}$. Find $f'(x)$

3. Let $f(x) = \sqrt{4 - x}$. Find $f'(2)$

Derivative Rules or "Short Cuts"

There are certain functions that you should know how to take the derivative of.
Make sure you have these memorized!

A CONSTANT: $\frac{d}{dx}(c) = 0$ [1]

Examples: $\frac{d}{dx}(42) =$ $\frac{d}{dx}(\sqrt{3}) =$ $\frac{d}{dx}(-\pi) =$

X TO A CONSTANT POWER: $\boxed{\frac{d}{dx}(x^n) = nx^{n-1}}$ [2]

Examples: $\frac{d}{dx}(x^5) =$ $\frac{d}{dx}\left(\frac{1}{x^3}\right) =$ $\frac{d}{dx}(\sqrt[3]{x}) =$

THE TRIG DERIVATIVES:

$\frac{d}{dx}(\sin x) =$ $\frac{d}{dx}(\cos x) =$

$\frac{d}{dx}(\sec x) =$ $\frac{d}{dx}(\csc x) =$

$\frac{d}{dx}(\tan x) =$ $\frac{d}{dx}(\cot x) =$

DERIVATIVES OF EXPONENTIAL FUNCTIONS:

$\frac{d}{dx}(e^x) =$ $\frac{d}{dx}(a^x) =$ $\frac{d}{dx}(x^x) =$

$\frac{d}{dx}(\ln x) =$ $\frac{d}{dx}(\log_b x) =$

DERIVATIVES OF INVERSE TRIG FUNCTIONS:

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}} \qquad (-1 < x < 1)$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2} \qquad (-\infty < x < \infty)$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}(\csc^{-1} x) = -\frac{1}{|x|\sqrt{x^2-1}} \qquad (x < -1, x > 1)$$

IMPORTANT: These rules ONLY work when for these exact functions with a single variable (i.e. x) plugged in to them!
What do we do if we can see that we have more than one of these basic functions to differentiate in the same problem? Well, that depends entirely on how the functions are combined.

[1] c is a constant (no x's allowed!)
[2] n is a constant (no x's allowed!)

Here are the ways the basic functions can be combined and what to do about it:

The functions are:

The constant multiple rule:

$\boxed{\left(c \cdot f(x)\right)' = c \cdot f'(x)}$ [3]

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The sum/difference rule:

$\boxed{(f \pm g)' = f' \pm g'}$

Example: If $y = 3x^4 - 6x^3 + 7x^{\frac{3}{2}}$, then $y' =$ ______________________________

The product rule:

$\boxed{(f \cdot g)' = f' \cdot g + f \cdot g'}$

Example: If $y = (3x^2 + 2x - 7)(4x^8 - 10x + 3)$, then $y' =$ ______________________________

The quotient rule:

$\boxed{\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}}$

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Example: Find $\frac{dy}{dx}$ if $y = \frac{x^2+3x-5}{x^4+9x}$.

[3] c is a constant (no x's allowed!)

The two functions are:

The Chain Rule:

$$\boxed{[f(g(x))]' = f'[g(x)] \cdot g'(x)}$$

Alternate Notation #1: $\frac{d}{dx}(f \circ g)(x) = f'\big(g(x)\big) \cdot g'(x)$

Alternate Notation #2: $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$, where $y = f(u)$ and $u = g(x)$

Derivative Rules Examples:

The best thing you can do now is practice finding lots of derivatives using all the rules. Find y' for the following.

Do not simplify the results.

1. $y = (x^5 - 3x)^{57}$

2. $y = e^{x\cot x}$

3. $y = \sec(x^3)$ vs. $y = \sec^3(x)$

4. $y = \sin\big(\sqrt{3x^4 - 5x}\big)$,

5. $y = \frac{2x-3}{(3x^2-4x)^3}$

6. $y = \left(\sqrt{3x^3 - 5x^2}\right)\sin(6x) + \sqrt{\pi + 4}$

7. $y = \cos^3\left(\frac{x^2}{1-x}\right)$

8. Let $f(2) = 6$, $f'(2) = -3$, $f'(8) = -7$, $f'(5) = 4$, $g(2) = 8$ and $g'(2) = 5$. If $h(x) = f\left(g(x)\right)$, find $h'(2)$.