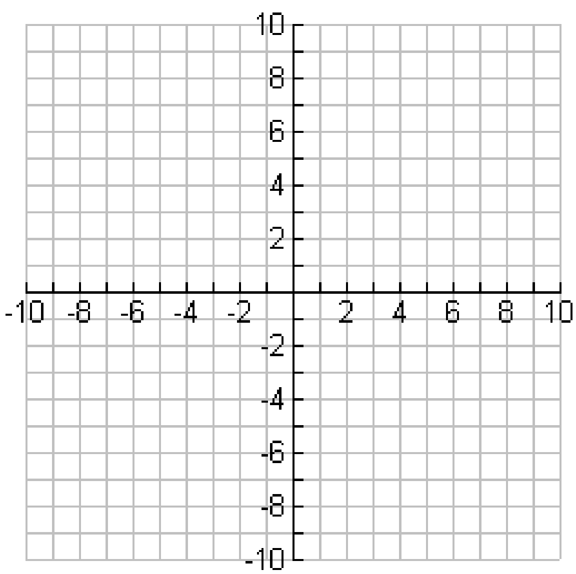
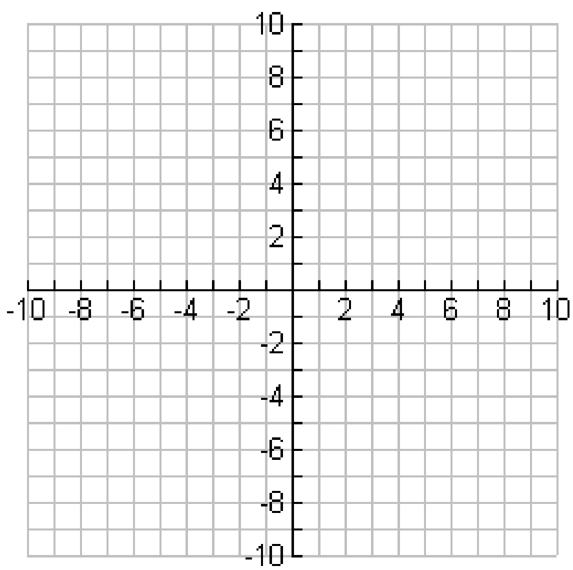
MSLC Workshop Series
Math 1151 – Workshop 1
Limits and Continuity

Warm-up:

Let $\mathrm{f}(x) = \frac{x^2-6x+8}{x-2}$ and let $\mathrm{g}(x) = \mathrm{x} - 4$. Are *f* and *g* equivalent functions? Why or why not? Hint: Try factoring the numerator of $\mathrm{f}(\mathrm{x})$.

Draw the graphs of $\mathrm{y} = \mathrm{f}(x)$ and $\mathrm{y} = \mathrm{g}(x)$.





Limits:

There is a technical definition, but for us, the intuitive definition will do just fine. Here it is:

The limit L is the value the function "gets close to" if we make the *x* values "get close to" (but not equal to) a. We write $\lim_{x \to a} f(x) = L$.

Continuity:

Definition: A function $f(x)$ is continuous at $x = a$ if and only if $\lim_{x \to a} f(x) = f(a)$.

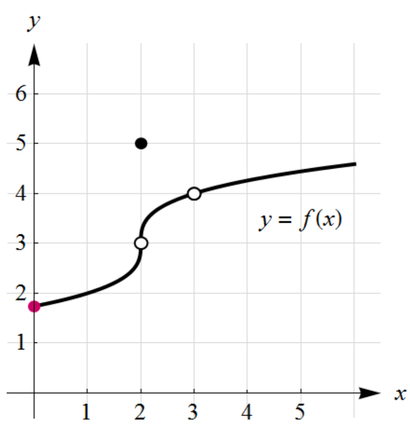
This definition actually states three things:

1.

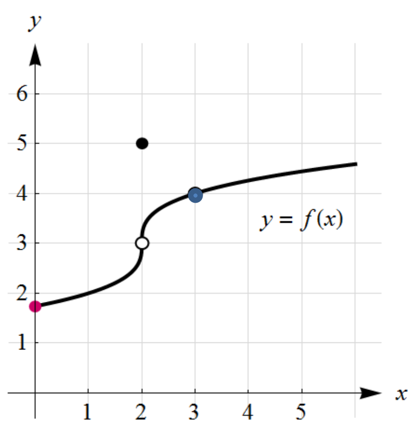
2.

3.

Limits

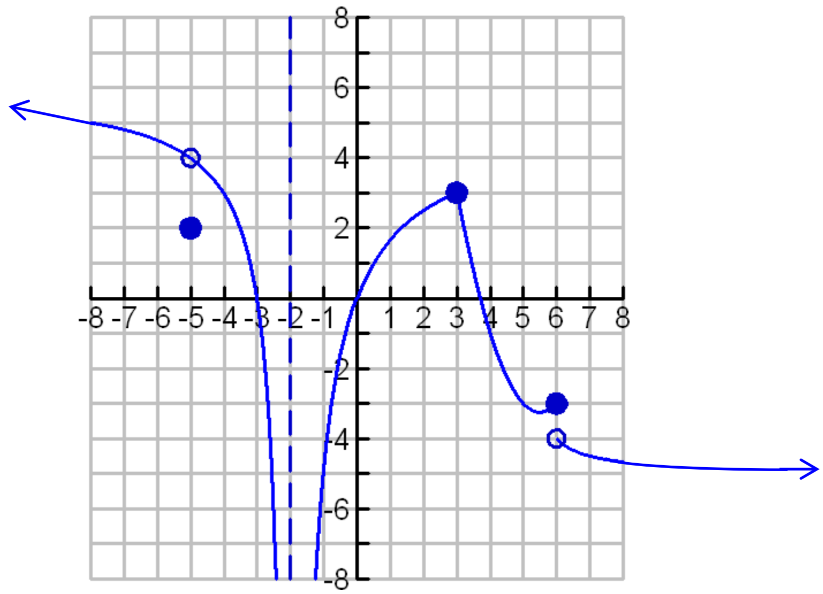


Continuity:



Part 1: Limits and Continuity given by a Graph

Let y = f(x) be given by the graph below. Use the graph to find the following.



1. a. $f(1) =$

   b. $\lim_{x \to 1} f(x) =$

   c. Is f continuous at $x = 1$?

   d. $\lim_{x \to 1} (f(x) + 3x) =$

2. a. $\lim_{x \to -2^-} f(x) =$

   b. $\lim_{x \to -2^+} f(x) =$

   c. $\lim_{x \to -2} f(x) =$

   d. Is f continuous at $x = -2$?

3. a. $\lim_{x \to -5^-} f(x) =$

   b. $\lim_{x \to -5^+} f(x) =$

   c. $\lim_{x \to -5} f(x) =$

   d. $f(-5) =$

   e. Is f continuous at $x = -5$?

   f. $\lim_{x \to -5} (4f(x)) =$

4. a. $\lim_{x \to 3^-} f(x) =$

   b. $\lim_{x \to 3^+} f(x) =$

   c. $\lim_{x \to 3} f(x) =$

   d. $f(3) =$

   e. Is f continuous at $x = 3$?

   f. $\lim_{x \to 3} \sin\left(\frac{\pi}{3} f(x)\right) =$

5. a. $\lim_{x \to 6^-} f(x) =$

   b. $\lim_{x \to 6^+} f(x) =$

   c. $\lim_{x \to 6} f(x) =$

   d. $f(6) =$

   e. Is f continuous at $x = 6$?

   f. $\lim_{x \to \infty} f(x) =$

   g. $\lim_{x \to -\infty} f(x) =$

Part 2: Limits and Continuity given by an Equation

Hints about finding Limits:

- Plugging in nearby numbers will get you no credit unless the directions specifically say to do so.
- For any function which is CONTINUOUS, you can find the limit just by plugging in the number as long as the answer is defined.
    - In particular we know the following functions and all their combinations are continuous wherever they are defined:

      ______________________________ ______________________________

      ______________________________ ______________________________

      ______________________________ ______________________________

      ____

    - Ways you can combine continuous functions to get another continuous function:

      ______________________________ ______________________________

      ______________________________ ______________________________

      ______________________________ ______________________________

- What do you do if the function is undefined? Look at the form of the limit!

$\frac{\text{zero}}{\text{nonzero number}}$ $\frac{\text{nonzero number}}{\text{zero}}$ $\frac{\text{zero}}{\text{zero}}$

$\frac{\text{number}}{\text{infinity}}$ $\frac{\text{infinity}}{\text{infinity}}$

Limits Problems:

a) $f(x) = \frac{1}{x-1}$

  a) $\lim_{x \to 1^-} f(x) =$

  b) $\lim_{x \to 1^+} f(x) =$

  c) $\lim_{x \to 1} f(x) =$

  d) $\lim_{x \to \infty} f(x) =$

  e) List the asymptotes of f. Justify your answers by citing appropriate limits. Explain how you know there are no others.

b) $f(x) = \frac{x^2-1}{x+1}$

  a) $\lim_{x \to -1^-} f(x) =$

  b) $\lim_{x \to -1^+} f(x) =$

  c) $\lim_{x \to -1} f(x) =$

  d) $\lim_{x \to \infty} f(x) =$

c) $h(x) = \frac{x^2-3x-4}{2x^2-4x-6} = \frac{(x-4)(x+1)}{2(x+1)(x-3)}$

  a) $\lim_{x \to -1} h(x) =$

  b) $\lim_{x \to 3} h(x) =$

  c) $\lim_{x \to 4} h(x) =$

d) $\lim_{x \to \infty} \mathrm{h}(x) =$

e) List the asymptotes of h. Justify your answers by citing appropriate limits. Explain how you know there are no others.

d) $\mathrm{h}(x) = \begin{Bmatrix} -x^2 + 9, & x \leq -2 \\ -2x + 1, & -2 < x < 2 \\ x + 1 & x \geq 2 \end{Bmatrix}$

a) $\lim_{x \to -2} \mathrm{h}(x) =$

b) $\lim_{x \to 2} \mathrm{h}(x) =$

c) $\lim_{x \to 1} \mathrm{h}(x) =$

e) $\lim_{x \to 0} \frac{x}{\sqrt{x+1} - 1}$

Comparison of Limits vs. Continuity

Limits

Conceptually

Where is the function headed (y-value) as you get near a certain x-value?

Graphically

No jumps or infinite squiggles, ignore the point itself

Algebraically

Limits from both sides have to agree

Math Notation

$$\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$$

$f(x)$ is defined on an interval on both sides of a

Continuity

Conceptually

Can you draw it without picking up your pencil?

Graphically

No holes, breaks or infinite squiggles

Algebraically

1. Limits from both sides have to agree
2. The y-value of the point has to agree with the limit

Math Notation

1. $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$
2. $f(\text{a})$ is defined
3. $\text{f}(a) = \lim_{x\to a} \text{f}(x)$

Squeeze Theorem

Let $f(x) = \sin(x)$. Evaluate the limit:

$$\lim_{x\to\infty} \frac{f(x+1)}{2x^2}$$

Intermediate Value Theorem

If f is a continuous function on the closed interval $[a, b]$, and d is any value between $f(a)$ and $f(b)$, then there is a number c in (a, b) such that $f(c) = d$.

We will use the IVT to show that the equation

$$2\log(x) = \frac{1}{\pi}$$

has a solution on the interval (1, 10).

a) IVT requires a single function, f. What is your choice for $\mathrm{f}(x)$? ____________________________

b) **On which interval** do we need to show that f is continuous? ______________________________

c) Explain why f is continuous on that interval.

d) Evaluate $\mathrm{f}(1)$ and $\mathrm{f}(10)$. $\mathrm{f}(1) =$

$\mathrm{f}(10) =$

e) IVT requires a number, d. What is your choice for d? $\mathrm{d} =$

f) Fill in the blanks appropriately (according to your answers above): $\mathrm{f}(\quad) < \mathrm{d} < \mathrm{f}(\quad)$

g) Based on your answers above, explain how to use IVT to determine that the equation has a solution in the interval (1, 10).