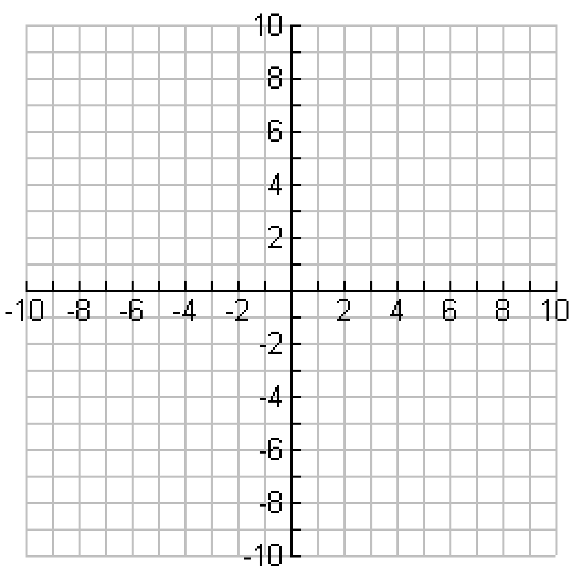
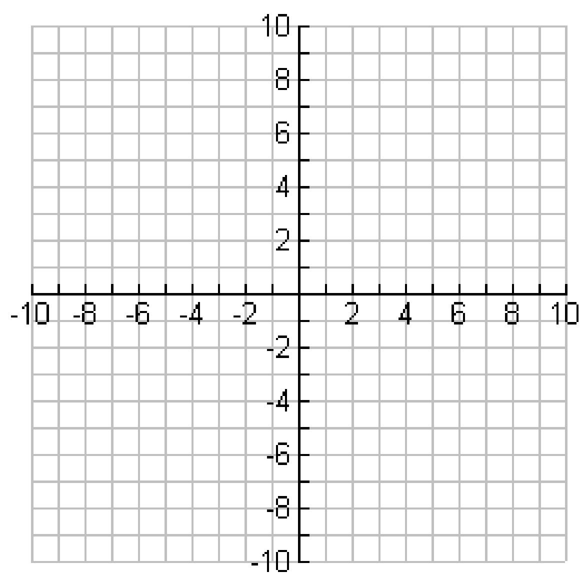
MSLC Workshop Series
Math 1151
Limits and Continuity

Warm-up: Let $f(x) = \frac{x^2-6x+8}{x-2}$ and let $g(x) = x - 4$. Are f and g equivalent functions? Why or why not?

Draw the graphs of $y = f(x)$ and $y = g(x)$.



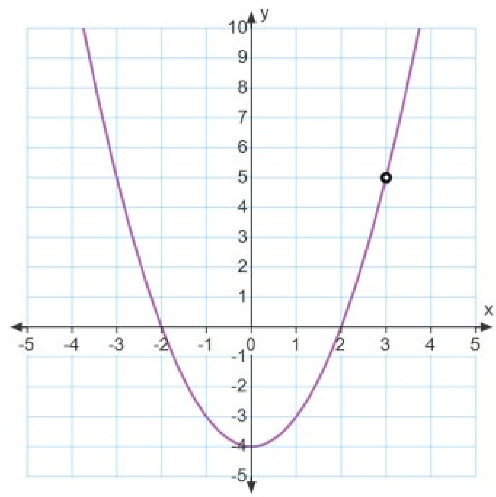


Limits:

There is a technical definition, but for us, the intuitive definition will do just fine. Here it is:

The limit *L* is the value the function "gets close to" if we make the *x* values "get close to" (but not equal to)

Quick Check: Find the value of the $\lim_{x \to 3} f(x)$ from the graph.



Continuity:

Def A function $f(x)$is continuous at $x = a$ if and only if $\lim_{x \to a} f(x) = f(a)$.

This definition actually states three things:

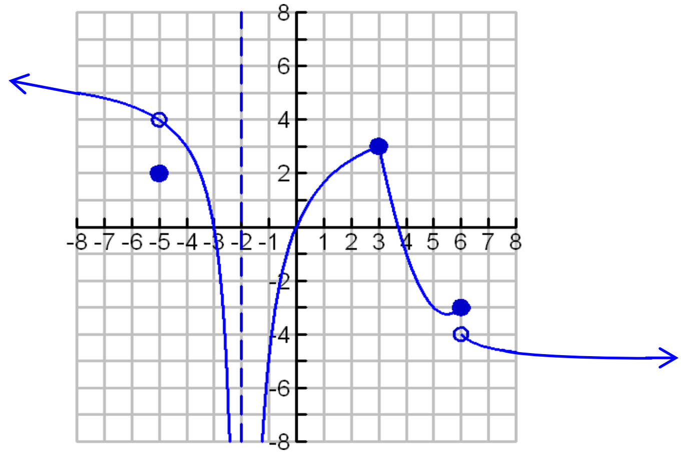
1.

2.

3.

Part 1: Limits and Continuity given by a Graph

Let $y = f(x)$ be given by the graph below. Use the graph to find the following.



1. a. $f(1) =$

   b. $\lim_{x\to 1} f(x) =$

   c. Is f continuous at $x = 1$?

2. a. $\lim_{x\to -2^-} f(x) =$

   b. $\lim_{x\to -2^+} f(x) =$

   c. $\lim_{x\to -2} f(x) =$

   d. Is f continuous at $x = -2$?

3. a. $\lim_{x\to -5^-} f(x) =$

   b. $\lim_{x\to -5^+} f(x) =$

   c. $\lim_{x\to -5} f(x) =$

   d. $f(-5) =$

   e. Is f continuous at $x = -5$?

4. a. $\lim_{x\to 3^-} f(x) =$

   b. $\lim_{x\to 3^+} f(x) =$

   c. $\lim_{x\to 3} f(x) =$

   d. $f(3) =$

   e. Is f continuous at $x = 3$?

5. a. $\lim_{x\to 6^-} f(x) =$

   b. $lim_{x\to 6^+} f(x) =$

   c. $lim_{x\to 6} f(x) =$

   d. $f(6) =$

   e. Is f continuous at $x = 6$?

Part 2: Limits, Continuity, and Differentiability given by an Equation

Hints about finding Limits:

- Plugging in nearby numbers will get you no credit unless the directions specifically say to do so.
- For any function which is CONTINUOUS, you can find the limit just by plugging in the number as long as the answer is defined.
    - In particular we know the following functions and all their combinations are continuous wherever they are defined:

      ______________________________ ______________________________

      ______________________________ ______________________________

      ______________________________ ______________________________

      ____

    - Ways you can combine continuous functions to get another continuous function:

      ______________________________ ______________________________

      ______________________________ ______________________________

      ______________________________ ______________________________

- What do you do if the function is undefined?

$\frac{\text{zero}}{\text{nonzero number}}$ $\frac{\text{nonzero number}}{\text{zero}}$ $\frac{\text{zero}}{\text{zero}}$

$\frac{\text{number}}{\text{infinity}}$ $\frac{\text{infinity}}{\text{infinity}}$

Limit Examples:

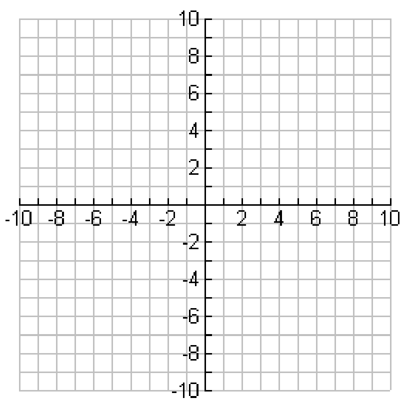
1. $\lim_{x \to 4} \frac{3-2x}{(x-4)(x+2)}$

2. $\lim_{x \to 4} \frac{x^2-4x}{x^2-3x-4}$

3. $\lim_{h \to 0} \frac{\sqrt{1+h}-1}{h}$

4. $f(x) = \begin{cases} 4 - x^2 & x \leq 2 \\ x - 1 & x > 2 \end{cases}$

   a. Find $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$.
   b. Does $\lim_{x \to 2} f(x)$ exist?
   c. Sketch a graph of f(x).



5. Challenge Problem! $\lim_{x \to -\infty} \frac{3x^3}{\sqrt{9x^6+x}}$

Continuity Examples: Determine if the following functions are continuous at the given x-value.

1. $f(x) = \begin{cases} 3x^2 + 7 & x < 2 \\ \sin(x) & x > 2 \end{cases}$, x=2

2. $g(x) = \begin{cases} \frac{x^2-2x-8}{x-4} & x \neq 4 \\ 7 & x = 4 \end{cases}$, x=4

Comparison of Limits vs. Continuity

Limits

Conceptually

Where is the function headed (y-value) as you get near a certain x-value?

Graphically

No jumps or infinite squiggles, ignore the point itself

Algebraically

Limits from both sides have to agree

Math Notation

$$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$$

$f(x)$ is defined on an interval on both sides of a

Continuity

Conceptually

Can you draw it without picking up your pencil?

Graphically

No holes, breaks or infinite squiggles

Algebraically

1. Limits from both sides have to agree
2. The y-value of the point has to agree with the limit

Math Notation

1. $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$
2. $f(\mathrm{a})$ is defined
3. $f(a) = \lim_{x \to a} f(x)$