MSLC Workshop Series
Math 1148 – 1150 Workshop: Combining Functions & Inverse Functions

The goal of this workshop is to familiarize you with various ways of combining functions together, and how to find the inverse of a function.

We will start by looking at how the four basic arithmetic operations apply to functions.

Adding, subtracting, multiplying, and dividing functions

These operations are used with functions just like they would be with numbers and variables. The only difference is the way these operations are indicated when they're used with functions.

$$(f+g)(x) = f(x) + g(x)$$
$$(f-g)(x) = f(x) - g(x)$$
$$(f \cdot g)(x) = f(x) \cdot g(x)$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Here are some examples for you to try.

Given $f(x) = \frac{1}{x}$ and $g(x) = x^2$ find:

$(f+g)(x)$ and its domain

$(f+g)(x) =$ ______________________

Domain of $(f+g)(x)$: ______________________

$(f-g)(x)$ and its domain

$(f-g)(x) =$ ______________________

Domain of $(f-g)(x)$: ______________________

$(f \cdot g)(x)$ and its domain

$(f \cdot g)(x) =$ ______________________

Domain of $(f \cdot g)(x)$: ______________________

$\left(\frac{f}{g}\right)(x)$ and its domain

$\left(\frac{f}{g}\right)(x) =$ ______________________

Domain of $\left(\frac{f}{g}\right)(x)$: ______________________

Composition of functions

This is another type of operation that can be performed on functions. It is a little different from the basic four arithmetic operations above, but it's carried out using substitution skills you have already seen in previous math classes.

Regular substitution example:

Given $f(x) = \frac{1}{2x+7}$, find $f(3)$.

$$f(3) = \frac{1}{2(3)+7}$$
$$= \frac{1}{6+7}$$
$$= \frac{1}{13}$$

Composition example:

Given $f(x) = \frac{1}{2x+7}$ and $g(x) = x^2 - 9$, find $(f \circ g)(x)$

$$(f \circ g)(x) = f\big(g(x)\big)$$
$$= f(x^2 - 9)$$
$$= \frac{1}{2(x^2-9)+7}$$
$$= \frac{1}{2x^2 - 18 + 7}$$
$$= \frac{1}{2x^2 - 11}$$

*NOTE: When finding the domain of a composition of functions, **always use the unsimplified form**. The simplification process can cause you to overlook important restrictions on domain.*

Here are a few examples for you to try.

Given $f(x) = x^2 - 3$ and $g(x) = \sqrt{x}$ find:

$(f \circ g)(x)$ and its domain in interval notation

$(f \circ g)(x) =$ ____________________

Domain of $(f \circ g)(x)$: ____________________

$(g \circ f)(x)$ and its domain in interval notation

$(g \circ f)(x) =$ ____________________

Domain of $(g \circ f)(x)$: ____________________

More composition examples

Given $f(x) = x^2 - 3$ and $g(x) = \sqrt{x}$ find:

$(f \circ f)(x)$ and its domain in interval notation

$(f \circ f)(x) =$ ____________________

Domain of $(f \circ f)(x)$: ____________________

$(g \circ g)(x)$ and its domain in interval notation

$(g \circ g)(x) =$ ____________________

Domain of $(g \circ g)(x)$: ____________________

Inverses of functions

When trying to find the inverse of a function, first determine if that function is one-to-one. The fastest way to determine whether or not a function is one-to-one is to use the horizontal line test on the graph of that function.

If the function isn't one-to-one, then either the restrictions on the domain that force the function to be one-to-one will need to be given or you will need to determine those restrictions on the domain yourself. (*If you are asked to determine the domain restrictions of a non-one-to-one function yourself, there will be more than one correct answer possible. In general, you will not be asked to determine domain restrictions for a non-one-to-one function on an exam for this course*.)

There are four major steps to find the inverse of a function.

1. Write the equation in terms of *x* and *y*.
2. Solve the equation for *x*.
3. Switch the *x*'s and *y*'s in the equation
4. Replace the *y* with inverse function notation.

Domain and Range with Inverse Functions

Keep in mind that the Domain of the original function is the Range of the inverse function, and vice versa. So Domain of $f(x) =$ Range of $f^{-1}(x)$ and Range of $f(x) =$ Domain of $f^{-1}(x)$.

Find the inverse function, as well as the domain and range of the following functions.

$f(x) = (x+5)^3 - 8$

$f^{-1}(x) =$ ____________________

Domain of $f(x)$: ____________________

Range of $f(x)$: ____________________

Domain of $f^{-1}(x)$: ____________________

Range of $f^{-1}(x)$: ____________________

$g(x) = \frac{3x}{x-2}$

$g^{-1}(x) =$ ____________________

Domain of $g(x)$: ____________________

Range of $g(x)$: ____________________

Domain of $g^{-1}(x)$: ____________________

Range of $g^{-1}(x)$: ____________________

$h(x) = 3 - \frac{x^4}{5};\ x \geq 0$

$h^{-1}(x) =$ ____________________

Domain of $h(x)$: ____________________

Range of $h(x)$: ____________________

Domain of $h^{-1}(x)$: ____________________

Range of $h^{-1}(x)$: ____________________