MSLC Workshop Series
Math 1130, 1148, and 1150 – Exponentials and Logarithms Workshop

First, a quick recap of what constitutes an exponential function.

An exponential function with a base of b is defined for all real numbers x by:

$$f(x) = b^x, \text{ where } b > 0 \text{ and } b \neq 1.$$

Essentially, this means an exponential function needs to have a positive number (that's not equal to 1) raised to a power that contains a variable.

Exponential functions might look a bit different than other functions you've encountered that have exponents, but they are still subject to the same rules for exponents.

<u>The Laws (or Rules) of Exponents</u>

*For all rules, we will assume that **a** and **b** are positive numbers.*

$$a^n \cdot a^m = a^{m+n}$$

$$(a^n)^m = a^{nm}$$

$$(a \cdot b)^k = a^k \cdot b^k$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

$$\frac{a^n}{a^m} = a^{n-m}$$

$$a^{n/m} = \sqrt[m]{a^n} = \left(\sqrt[m]{a}\right)^n$$

Exponential Review Problems: True or False?

a. $2^4 \cdot 3^5 = 6^9$

b. $2^5 \cdot 3^5 = 6^5$

c. $\left(\frac{3^{-3}}{2^{-2}}\right)^{-1} = \frac{4}{27}$

d. $(2^3 \cdot 3^2)^{\frac{1}{5}} = 6^{\frac{6}{5}}$

e. $\sqrt[4]{\frac{3^8}{2^{12}}} = \frac{3^2}{2^3}$

f. $-5 \cdot 2^{-3} = -\frac{5}{8}$

Now for a quick review of what constitutes a logarithmic function.

A logarithm with a base of a positive number b is defined to be:

$$log_b(x) = y \;\Leftrightarrow\; b^y = x$$

*Remember, a logarithmic function is the inverse of an exponential function. The answer to $log_b(x)$ gives you the exponent that **b** needs to be raised to in order to get an answer of **x**.*

The Rules for Logarithms

*For all rules, we will assume that **a, b, A, B,** and **C** are positive numbers.*

Definition of a logarithm:

$$\boldsymbol{log}_b(x) = y \;\Leftrightarrow\; b^y = x$$

Useful properties of logarithms:

$$\boldsymbol{log}_a(A \cdot B) = \boldsymbol{log}_a A + \boldsymbol{log}_a B \qquad \boldsymbol{log}_a\left(\frac{B}{C}\right) = \boldsymbol{log}_a B - \boldsymbol{log}_a C$$

$$\boldsymbol{log}_a(A^n) = n \cdot \boldsymbol{log}_a A$$

Cancelation Rules:

$$\boldsymbol{log}_a(a^n) = n \qquad a^{\boldsymbol{log}_a n} = n$$

Change of Base Formula:

$$\boldsymbol{log}_a(C) = \frac{\boldsymbol{log}_b(C)}{\boldsymbol{log}_b(a)}$$

Logarithm Review Problems

1. True or False?

   a. $ln(A+B) = ln\,A + ln\,B$

   b. $ln\,e + ln\,e^2 = 3$

   c. $log((-10)^2) = 2$

   d. $\frac{log\,A}{log\,B} = log\,A - log\,B$

   e. $log(-10) + log(-10) = 2$

   f. $log(0.1) = -1$

Logarithm Review Problems (continued)

2. Use the properties of logarithms to rewrite the logarithm as a sum or difference of logarithms. Express the exponents as factors.

$$ln\left(\frac{(y-4)^6}{7\sqrt{y^2-81}}\right)$$

3. Use the properties of logarithms to rewrite the following expressions a single logarithm.
$\frac{4}{3}log_{\pi} x^9 + log_{\pi}(x-2) - \frac{1}{2}[8\, log_{\pi}(x-2) - log_{\pi}(x+5)^6]$

Tips of solving Exponential or Logarithmic Equations

- Don't forget previously learned items such as factoring and other basic algebraic techniques for solving equations.

- For equations containing exponents, logarithms may only be necessary if the variable is in the exponent.

- For equations containing logarithms, properties of logarithms may not always be helpful unless the variable is inside the logarithm.

Solve the following exponential equations

1. $5(2^x) = e^{3x+1}$

2. $\sqrt[4]{x^3 - 35} - 2 = 4$

3. $7^{2n} - 7^{n+1} + 10 = 0$

Solve the following logarithmic equations

1. $log_m(16807) = 5 + log_m(7776)$

2. $log\, a + log(a - 2) = log\, 2\,4$

3. $log\, 7^w + 29 = ln\, \pi - 8ew$

4. $log_3(c + 15) - log_3(1 - c) = 2$

Modeling with Exponential and Logarithmic Functions

Modeling problems are not significantly different from the more straightforward exponential and logarithmic equations we've seen so far.

The primary difference is that the modeling problem will likely be presented in multiple parts. Each part of the modeling problem will ask for something that you've already seen as a stand-alone problem

Radioactive Decay

Doctors use radioactive iodine to help diagnose some thyroid disorders. This iodine isotope decays according to the function: $a(t) = 14e^{-.087t}$ where a is the amount of iodine remaining (in grams) after t days.

a. Find the amount of radioactive iodine originally present (i.e. when $t = 0$).

b. How much radioactive iodine is left after 20 days?

c. How long will it take for half of the original radioactive iodine to remain?

Compound Interest

How long will it take an investment of $100 to double if it is invested at:

a. 8% compounded quarterly

b. 6% compounded monthly

c. 7% compounded continuously

Population Growth **(*<u>Challenging</u>*)**

The number of bacteria in a colony grows exponentially over time. If there are 75 bacteria present after 2 hours and 9375 bacteria present after 5 hours:

a. Use the formula $n(t) = n_0 e^{r \cdot t}$ to determine the formula for the number of bacteria in the colony, *n,* present after *t* hours. Assume n_0 represents the number of bacteria present initially.

b. How many bacteria will be present in the colony after 10 hours?