Transformations of Graphs

Vertical Shifts

\[ y = f(x) + k \]
Shifts graph \( k \) units up
(Add \( k \) to \( y \)-coordinate)

\[ y = f(x) - k \]
Shifts graph \( k \) units down
(Subtract \( k \) from \( y \)-coordinate)

Horizontal Shifts

\[ y = f(x - h) \]
Shifts graph \( h \) units to the right
(Add \( h \) to \( x \)-coordinate)

\[ y = f(x + h) \]
Shifts graph \( h \) units to the left
(Subtract \( h \) from \( x \)-coordinate)

Stretching and Compressing

Vertical

\[ y = c \cdot f(x), \text{ where } c > 1 \]
Stretches graph vertically away from \( x \)-axis by a factor of \( c \)
(Multiply \( y \)-coordinate by \( c \))

\[ y = c \cdot f(x), \text{ where } 0 < c < 1 \]
Compresses graph vertically towards \( x \)-axis by a factor of \( c \)
(Multiply \( y \)-coordinate by \( c \))

Horizontal

\[ y = f(c \cdot x), \text{ where } 0 < c < 1 \]
Stretches graph horizontally away from \( y \)-axis by a factor of \( \frac{1}{c} \)
(Multiply \( x \)-coordinate by \( \frac{1}{c} \); i.e. the reciprocal of \( c \))

\[ y = f(c \cdot x), \text{ where } c > 1 \]
Compresses graph horizontally towards \( y \)-axis by a factor of \( \frac{1}{c} \)
(Multiply \( x \)-coordinate by \( \frac{1}{c} \); i.e. the reciprocal of \( c \))

Reflections

\[ y = -f(x) \]
Reflects graph about the \( x \)-axis
(Multiply \( y \)-coordinate by \(-1\))

\[ y = f(-x) \]
Reflects graph about the \( y \)-axis
(Multiply \( x \)-coordinate by \(-1\))