MSLC Workshop Series
Math 1149 & 1150 Workshop: Graphs of Trigonometric Functions

Remember:

1. The basic graphs of sine and cosine have a period of 2π
2. Changes in amplitude and period as well as phase shifts are nothing more than transformations you've seen before; they have just been given new names for trig functions.
   - *Changes in amplitude are vertical stretches or shrinks/compressions*
   - *Changes in period are horizontal stretches or shrinks/compressions*
   - *Phase shifts are horizontal (left or right) shifts*
   - *These graphs can also be shifted vertically, but that isn't shown in all classes.*

Graphs of Sine and Cosine

For the graphs of $y = A\sin(Bx - C) + D$ and $y = A\cos(Bx - C) + D$

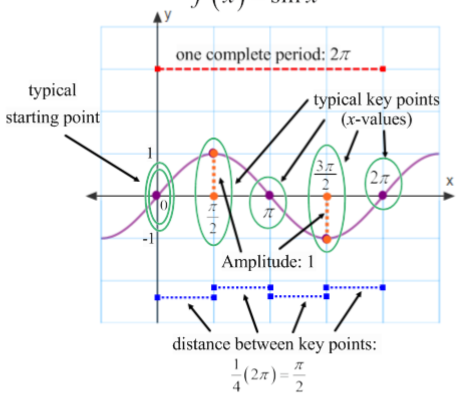
- Amplitude $= |A|$
- Period $= \frac{2\pi}{B}$
- Phase Shift $= \frac{C}{B}$
- Vertical Shift $= D$
- Distance Between Key Points* $= \left(\frac{1}{4}\right) \cdot$ (period)
- Reflections: Determined by the function used and the values of A and B.
  - For Sine functions – there will be a reflection if the leading coefficient (A) is negative, or if the coefficient of the x-term (B) is negative. If both are negative, there will not be a reflection.
  - For Cosine functions – there will only be a reflection if the leading coefficient (A) is negative. If the coefficient of the x-term (B) is negative, there is some algebraic simplification that must be done, but this will not result in a reflection.

**Key Points* are the points that are at the top or bottom of the graph, or the points on the center-line of the graph

Here are examples of a single change to each of these elements for the basic sine graph.

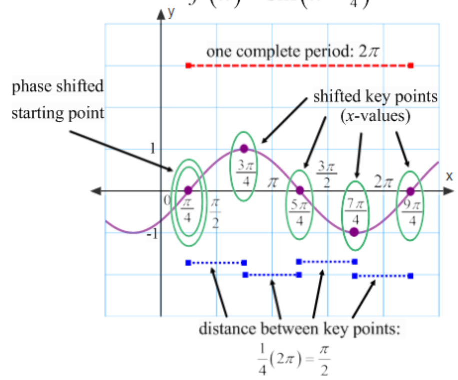
Basic Sine Graph

$f(x) = \sin x$



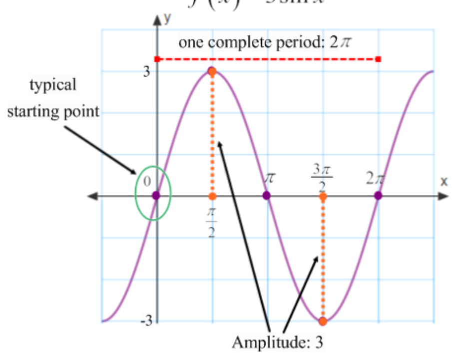
Phase Shift

$f(x) = \sin\left(x - \frac{\pi}{4}\right)$



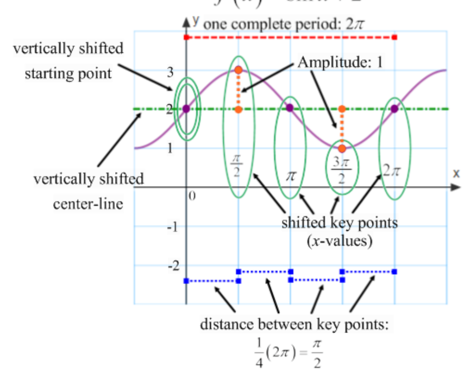
Change in Amplitude

$f(x) = 3\sin x$



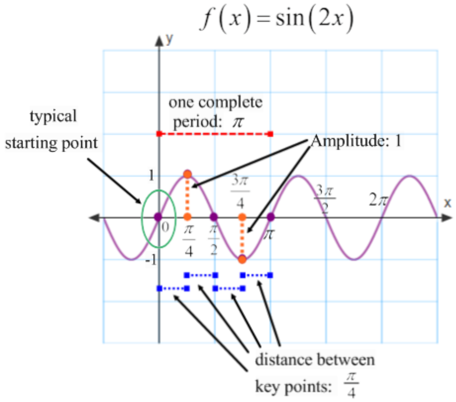
Vertical Shift

$f(x) = \sin x + 2$



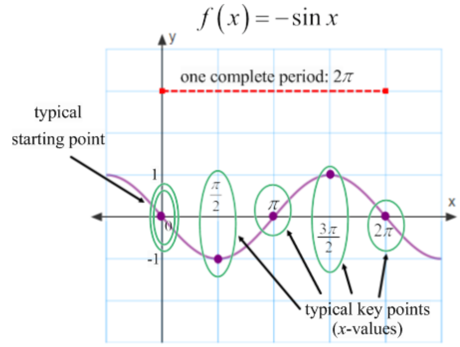
Change in Period

$f(x) = \sin(2x)$



Reflection

$f(x) = -\sin x$



Putting It All Together

First go through and find the amplitude, period, phase shift, vertical shift, reflection, and distance between key points for the equation you are trying to graph. Once all of the relevant pieces have been found, set up your graph by drawing the *x*- and *y*-axis and marking your maximum and minimum *y*-values based on your amplitude and any vertical shifts that may occur. Next, use the phase shift to determine the starting *x*-value of the period, then add the distance between key points to that starting point to find the next key point's *x*-value. Repeat this process until one complete period of *x*-values has been determined. This will be a ***total of five key points***: ***the starting point*** (determined by the phase shift) ***and four other key points*** (high points, low points, or center points).

Example:

Find the amplitude, period, phase shift, vertical shift, and any reflection of $y = -3\sin\left(\frac{\pi}{2}x + \frac{\pi}{4}\right) - 2$, then graph one complete period.

Amplitude: $|-3| = 3$

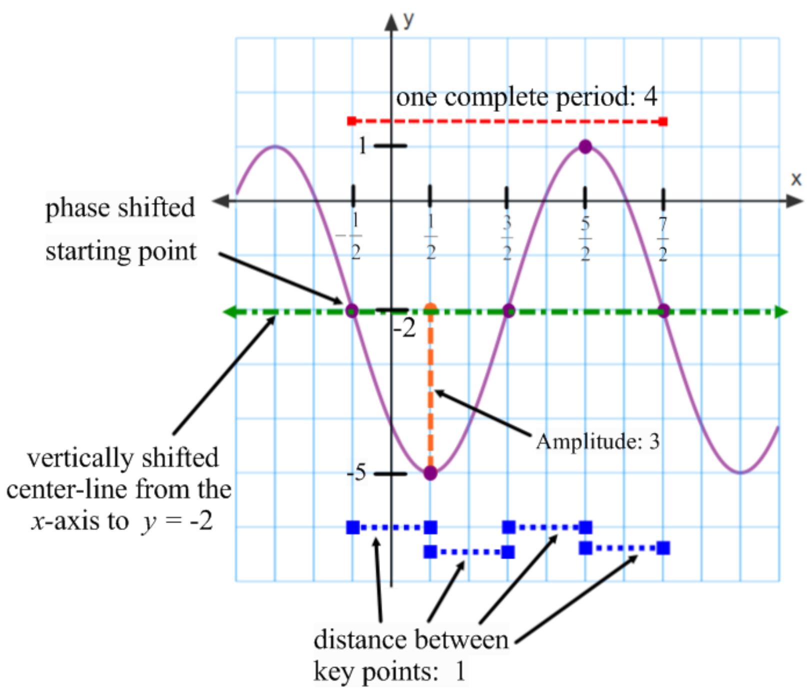
Period: $\dfrac{2\pi}{\frac{\pi}{2}} = 2\pi \cdot \dfrac{2}{\pi} = 4$

Phase Shift: $-\dfrac{\frac{\pi}{4}}{\frac{\pi}{2}} = -\dfrac{\pi}{4} \cdot \dfrac{2}{\pi} = -\dfrac{1}{2}$

Reflection: yes $(x - \text{axis})$

Vertical Shift: -2

Distance Between Key Points: $\frac{1}{4} \cdot (\text{Period}) = \frac{1}{4} \cdot 4 = 1$



1. Find the amplitude and period as well as any phase-shift, reflections, and vertical shifts of the following trigonometric equations. Then use that information to graph one complete period.

a) $y = 3\sin(6\pi x)$

Amplitude:

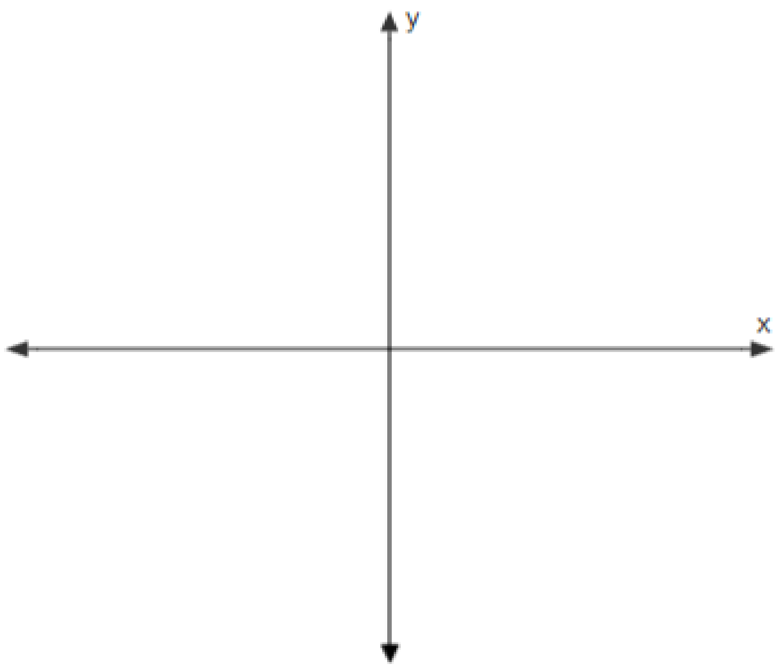
Period:

Phase Shift:

Reflection:

Vertical Shift:

Distance Between
Key Points:



b) $y = -2\cos 4\left(x + \frac{\pi}{4}\right)$

Amplitude:

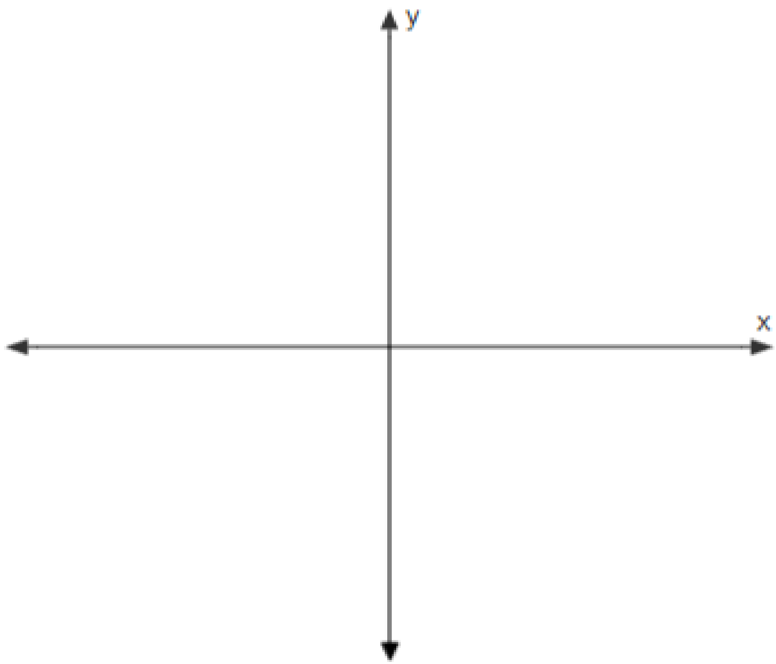
Period:

Phase Shift:

Reflection:

Vertical Shift:

Distance Between
Key Points:



1 (cont'd). Find the amplitude and period as well as any phase-shift, reflections, and vertical shifts of the following trigonometric equations. Then use that information to graph one complete period.

c) $y = 4\sin\left(-\frac{2}{5}x + \frac{6\pi}{25}\right)$

Amplitude:

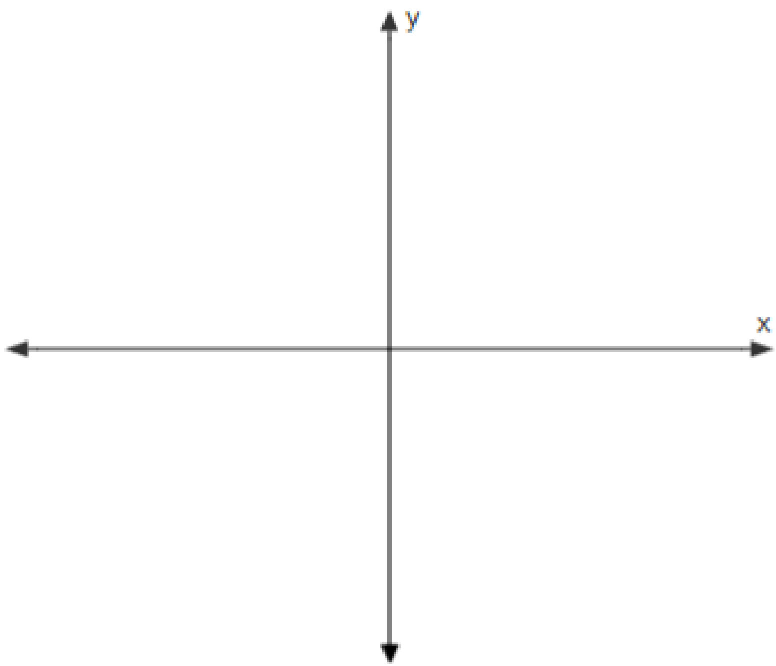
Period:

Phase Shift:

Reflection:

Vertical Shift:

Distance Between
Key Points:



d) $y = 8\cos\left(-4\pi x - \frac{5\pi}{7}\right) + 6$

Amplitude:

Period:

Phase Shift:

Reflection:

Vertical Shift:

Distance Between
Key Points: