<u>**Integral Calculus Formula Sheet**</u>

Derivative Rules:

$\boldsymbol{\frac{d}{dx}(c) = 0}$	$\frac{d}{dx}(x^n) = nx^{n-1}$	
$\frac{d}{dx}(\sin x) = \cos x$ $\frac{d}{dx}(\sec x) = \sec x \tan x$ $\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\cos x) = -\sin x$ $\frac{d}{dx}(\csc x) = -\csc x \cot x$ $\frac{d}{dx}(\cot x) = -\csc^2 x$	$\frac{d}{dx}(a^x) = a^x \ln a$ $\frac{d}{dx}(e^x) = e^x$
$\frac{d}{dx}\left(cf(x)\right) = c\frac{d}{dx}\left(f(x)\right)$	$\frac{d}{dx}\left(f(x) \pm g(x)\right) = \frac{d}{dx}\left(f(x)\right) \pm \frac{d}{dx}\left(g(x)\right)$	
$(f \cdot g)' = f' \cdot g + f \cdot g'$	$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$	$\boldsymbol{\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))g'(x)}$

Properties of Integrals:

$\int kf(u)du = k\int f(u)du$	$\int [f(u) \pm g(u)]\, du = \int f(u)du \pm \int g(u)du$
$\int_a^a f(x)dx = 0$	$\int_a^b f(x)dx = -\int_b^a f(x)dx$
$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$	$f_{ave} = \frac{1}{b-a}\int_a^b f(x)dx$
$\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$ if f(x) is even	$\int_{-a}^{a} f(x)dx = 0$ if f(x) is odd
$\int_a^b g(f(x))f'(x)dx = \int_{f(a)}^{f(b)} g(u)du$	$\int udv = uv - \int vdu$

Integration Rules:

$\int du = u + C$ $\int u^n\, du = \frac{u^{n+1}}{n+1} + C$ $\int \frac{du}{u} = \ln\lvert u\rvert + C$ $\int e^u du = e^u + C$ $\int a^u du = \frac{1}{\ln a}a^u + C$	$\int \sin u \;\; du = -\cos u + C$ $\int \cos u \;\; du = \sin u + C$ $\int \sec^2 u \;\; du = \tan u + C$ $\int \csc^2 u = -\cot u + C$ $\int \csc u \cot u \;\; du = -\csc u + C$ $\int \sec u \tan u \;\; du = \sec u + C$	$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C$ $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\left(\frac{u}{a}\right) + C$ $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\left(\frac{\lvert u\rvert}{a}\right) + C$

Fundamental Theorem of Calculus:

$F'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$ where $f(t)$ is a continuous function on [a, x].
$\int_a^b f(x)dx = F(b) - F(a)$, where F(x) is any antiderivative of f(x).

Riemann Sums:

$\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$ $\sum_{i=1}^{n} a_i + b_i = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$	$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(a + i\Delta x)\Delta x$ $\Delta x = \frac{b-a}{n}$
$\sum_{i=1}^{n} 1 = n$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$	$\sum_{i}$(height of ith rectangle) $\cdot$ (width of ith rectangle) Right Endpoint Rule: $\sum_{i=1}^{n} f(a + i\Delta x)\,(\Delta x) = \sum_{i=1}^{n} (\frac{(b-a)}{n}) f(a + i\frac{(b-a)}{n})$ Left Endpoint Rule: $\sum_{i=1}^{n} f(a + (i-1)\Delta x)\,(\Delta x) = \sum_{i=1}^{n} (\frac{(b-a)}{n}) f(a + (i-1)\frac{(b-a)}{n})$ Midpoint Rule: $\sum_{i=1}^{n} f(a + \left(\frac{(i-1)+i}{2}\right)\Delta x)\,(\Delta x) = \sum_{i=1}^{n} (\frac{(b-a)}{n}) f(a + \left(\frac{(i-1)+i}{2}\right)\frac{(b-a)}{n})$

Net Change:

Displacement: $\int_a^b v(x)dx$	Distance Traveled: $\int_a^b \lvert v(x)\rvert dx$	$s(t) = s(0) + \int_0^t v(x)dx$	$Q(t) = Q(0) + \int_0^t Q'(x)dx$

Trig Formulas:

$sin^2(\mathrm{x}) = \frac{1}{2}(1 - cos(2x))$	$tan\,x = \frac{sin\,x}{cos\,x}$	$sec\,x = \frac{1}{cos\,x}$	$cos(-\mathrm{x}) = cos(\mathrm{x})$	$sin^2(\mathrm{x}) + cos^2(\mathrm{x}) = 1$
$cos^2(\mathrm{x}) = \frac{1}{2}(1 + cos(2x))$	$cot\,x = \frac{cos\,x}{sin\,x}$	$csc\,x = \frac{1}{sin\,x}$	$\sin(-\mathrm{x}) = -\sin(\mathrm{x})$	$tan^2(\mathrm{x}) + 1 = sec^2(\mathrm{x})$

Geometry Fomulas:

Area of a Square: $A = s^2$	Area of a Triangle: $A = \frac{1}{2}bh$	Area of an Equilateral Trangle: $A = \frac{\sqrt{3}}{4}s^2$	Area of a Circle: $A = \pi r^2$	Area of a Rectangle: $A = bh$

Areas and Volumes:

Area in terms of *x* (vertical rectangles): $$\int_a^b (top - bottom)dx$$	Area in terms of *y* (horizontal rectangles): $$\int_c^d (right - left)dy$$
General Volumes by Slicing: Given: Base and shape of Cross-sections $V = \int_a^b A(x)dx$ if slices are vertical $V = \int_c^d A(y)dy$ if slices are horizontal	Disk Method: For volumes of revolution laying on the axis with slices perpendicular to the axis $V = \int_a^b \pi[R(x)]^2 dx$ if slices are vertical $V = \int_c^d \pi[R(y)]^2 dy$ if slices are horizontal
Washer Method: For volumes of revolution not laying on the axis with slices perpendicular to the axis $V = \int_a^b \pi[R(x)]^2 - \pi[r(x)]^2 dx$ if slices are vertical $V = \int_c^d \pi[R(y)]^2 - \pi[r(y)]^2 dy$ if slices are horizontal	Shell Method: For volumes of revolution with slices parallel to the axis $V = \int_a^b 2\pi rh dx$ if slices are vertical $V = \int_c^d 2\pi rh dy$ if slices are horizontal

Physical Applications:

Physics Formulas	Associated Calculus Problems
Mass: Mass = Density * Volume *(for 3-D objects)* Mass = Density * Area *(for 2-D objects)* Mass = Density * Length *(for 1-D objects)*	Mass of a one-dimensional object with variable linear density: $$Mass = \int_a^b (linear\ density) \underbrace{dx}_{distance} = \int_a^b \rho(x)dx$$
Work: Work = Force * Distance Work = Mass * Gravity * Distance Work = Volume * Density * Gravity * Distance	Work to stretch or compress a spring (force varies): $$Work = \int_a^b (force)dx = \int_a^b F(x)dx = \int_a^b \underbrace{kx}_{\substack{Hooke's\ Law \\ for\ springs}} dx$$ Work to lift liquid: $$Work = \int_c^d (gravity)(density)(distance) \underbrace{(area\ of\ a\ slice)dy}_{volume}$$ $$W = \int_c^d 9.8 * \rho * A(y) * (a - y)dy\ (in\ metric)$$
Force/Pressure: Force = Pressure * Area Pressure = Density * Gravity * Depth	Force of water pressure on a vertical surface: $$Force = \int_c^d (gravity)(density)(depth) \underbrace{(width)dy}_{area}$$ $$F = \int_c^d 9.8 * \rho * (a - y) * w(y)dy\ (in\ metric)$$

Integration by Parts:

Knowing which function to call *u* and which to call *dv* takes some practice. Here is a general guide:

u ↑	**I**nverse Trig Function	$(sin^{-1}x, arccos\, x$,etc)
	Logarithmic Functions	$(log\ 3x, ln(x+1)$,etc)
	Algebraic Functions	$(x^3, x+5, 1/x$,etc)
	Trig Functions	$(sin(5x), tan(x)$,etc)
↓ dv	**E**xponential Functions	$(e^{3x}, 5^{3x}$,etc)

Functions that appear at the top of the list are more like to be *u,* functions at the bottom of the list are more like to be *dv.*

Trig Integrals:

Integrals involving sin(x) and cos(x):	Integrals involving sec(x) and tan(x):
1. If the power of the sine is odd and positive: **Goal:** $u = cos\, x$ i. Save a $du = sin(x)dx$ ii. Convert the remaining factors to $cos(x)$ (using $sin^2 x = 1 - cos^2 x$.)	1. If the power of $sec(x)$ is even and positive: **Goal:**$u = tan\, x$ i. Save a $du = sec^2(x)dx$ ii. Convert the remaining factors to $tan(x)$ (using $sec^2 x = 1 + tan^2 x$.)
2. If the power of the cosine is odd and positive: **Goal:**$u = sin\, x$ i. Save a $du = cos(x)dx$ ii. Convert the remaining factors to $sin(x)$ (using $cos^2 x = 1 - sin^2 x$.)	2. If the power of $tan(x)$ is odd and positive: **Goal:**$u = sec(x)$ i. Save a $du = sec(x)\,tan(x)dx$ ii. Convert the remaining factors to $sec(x)$ (using $sec^2(x) - 1 = tan^2(x)$.)
3. If both $sin(x)$ and $cos(x)$ have even powers: Use the half angle identities: i. $sin^2(x) = \frac{1}{2}(1 - cos(2x))$ ii. $cos^2(x) = \frac{1}{2}(1 + cos(2x))$	• If there are no sec(x) factors and the power of tan(x) is even and positive, use $sec^2(x) - 1 = tan^2(x)$ to convert one $tan^2(x)$ to $sec^2(x)$ • Rules for sec(x) and tan(x) also work for csc(x) and cot(x) with appropriate negative signs
If nothing else works, convert everything to sines and cosines.	*If nothing else works, convert everything to sines and cosines.*

Trig Substitution:

Expression	Substitution	Domain	Simplification
$\sqrt{a^2 - u^2}$	$u = a\, sin\, \theta$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$	$\sqrt{a^2 - u^2} = a\, cos\, \theta$
$\sqrt{a^2 + u^2}$	$u = a\, tan\, \theta$	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$\sqrt{a^2 + u^2} = a\, sec\, \theta$
$\sqrt{u^2 - a^2}$	$u = a\, sec\, \theta$	$0 \le \theta \le \pi, \theta \ne \frac{\pi}{2}$	$\sqrt{u^2 - a^2} = a\, tan\, \theta$

Partial Fractions:

Linear factors:	Irreducible quadratic factors:
$\frac{P(x)}{(x-r_1)^m} = \frac{A}{(x-r_1)} + \frac{B}{(x-r_1)^2} + \ldots + \frac{Y}{(x-r_1)^{m-1}} + \frac{Z}{(x-r_1)^m}$	$\frac{P(x)}{(x^2+r_1)^m} = \frac{Ax+B}{(x^2+r_1)} + \frac{Cx+D}{(x^2+r_1)^2} + \ldots + \frac{Wx+X}{(x^2+r_1)^{m-1}} + \frac{Yx+Z}{(x^2+r_1)^m}$
If the fraction has multiple factors in the denominator, we just add the decompositions.	*If the fraction has multiple factors in the denominator, we just add the decompositions.*