# Integral Calculus Formula Sheet

## Derivative Rules:

<table>
<thead>
<tr>
<th>d/dx(c)</th>
<th>d/dx(x^n)</th>
<th>d/dx(sinx)</th>
<th>d/dx(cosx)</th>
<th>d/dx(secx)</th>
<th>d/dx(cscx)</th>
<th>d/dx(tanx)</th>
<th>d/dx(cotx)</th>
<th>d/dx(ax^n)</th>
<th>d/dx(e^x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>nx^n-1</td>
<td>cos x</td>
<td>-sin x</td>
<td>sec x tan x</td>
<td>-csc x cot x</td>
<td>sec^2 x</td>
<td>-csc^2 x</td>
<td>a^n x^n</td>
<td>e^x</td>
</tr>
</tbody>
</table>

## Integration Rules:

\[
\int k \, f(u) \, du = k \int f(u) \, du \\
\int_a^b f(x) \, dx = 0 \\
\int_a^b f(x) \, dx = -\int_a^b f(x) \, dx \\
\int_a^c f(x) \, dx = \int_a^b f(x) \, dx + \int_b^c f(x) \, dx \\
\int_{-a}^a f(x) \, dx = 2 \int_0^a f(x) \, dx \quad \text{if } f(x) \text{ is even} \\
\int_{-a}^a f(x) \, dx = 0 \quad \text{if } f(x) \text{ is odd} \\
\int_a^b g(f(x)) f'(x) \, dx = \left[ g(u) \right]_{f(a)}^{f(b)} \\
\int u \, dv = uv - \int v \, du
\]

## Properties of Integrals:

\[
\int [f(u) \pm g(u)] \, du = \int f(u) \, du \pm \int g(u) \, du \\
\int_a^b f(x) \, dx = -\int_a^b f(x) \, dx \\
\text{fave} = \frac{1}{b-a} \int_a^b f(x) \, dx \\
\text{If } f(x) \text{ is even, } \int_{-a}^a f(x) \, dx = 2 \int_0^a f(x) \, dx \\
\text{If } f(x) \text{ is odd, } \int_{-a}^a f(x) \, dx = 0
\]
Fundamental Theorem of Calculus:

\[
F'(x) = \frac{d}{dx} \int_a^x f(t) \, dt = f(x) \text{ where } f(t) \text{ is a continuous function on } [a, x].
\]

\[
\int_a^b f(x) \, dx = F(b) - F(a), \text{ where } F(x) \text{ is any antiderivative of } f(x).
\]

Riemann Sums:

\[
\sum_{i=1}^{n} \left( \begin{array}{c}
\text{height of } i\text{th rectangle}) \\
\text{· (width of } i\text{th rectangle)}
\end{array} \right) = \int_a^b f(x) \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(a + i \Delta x) \Delta x
\]

\[
\Delta x = \frac{b - a}{n}
\]

\[
\sum_{i=1}^{n} a_i = c \sum_{i=1}^{n} a_i
\]

\[
\sum_{i=1}^{n} b_i = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i
\]

\[
\sum_{i=1}^{n} 1 = n
\]

\[
\sum_{i=1}^{n} \frac{i}{1} = \frac{n(n + 1)}{2}
\]

\[
\sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6}
\]

\[
\sum_{i=1}^{n} i^3 = \left[ \frac{n(n + 1)}{2} \right]^2
\]

Net Change:

Displacement: \( \int_a^b v(x) \, dx \)

Distance Traveled: \( \int_a^b |v(x)| \, dx \)

\[
s(t) = s(0) + \int_0^t v(x) \, dx
\]

\[
Q(t) = Q(0) + \int_0^t Q'(x) \, dx
\]

Trig Formulas:

\[
sin^2(x) = \frac{1}{2} (1 - \cos(2x))
\]

\[
tan x = \frac{\sin x}{\cos x}
\]

\[
sec x = \frac{1}{\cos x}
\]

\[
\cos(-x) = \cos(x)
\]

\[
\sin(-x) = -\sin(x)
\]

\[
tan^2(x) + 1 = sec^2(x)
\]

\[
cos^2(x) = \frac{1}{2} (1 + \cos(2x))
\]

\[
cot x = \frac{\cos x}{\sin x}
\]

\[
csc x = \frac{1}{\sin x}
\]

\[
\sin(x) + \cos^2(x) = 1
\]

\[
\cos^2(x) + \sin^2(x) = 1
\]

\[
\tan^2(x) + 1 = \sec^2(x)
\]

Geometry Formulas:

<table>
<thead>
<tr>
<th>Area of a Square:</th>
<th>Area of a Triangle:</th>
<th>Area of an Equilateral Triangle:</th>
<th>Area of a Circle:</th>
<th>Area of a Rectangle:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A = s^2 )</td>
<td>( A = \frac{1}{2} bh )</td>
<td>( A = \frac{\sqrt{3}}{4} s^2 )</td>
<td>( A = \pi r^2 )</td>
<td>( A = bh )</td>
</tr>
</tbody>
</table>
## Areas and Volumes:

### Area in terms of $x$ (vertical rectangles):

$$
\int_a^b (\text{top} - \text{bottom})dx
$$

### Area in terms of $y$ (horizontal rectangles):

$$
\int_c^d (\text{right} - \text{left})dy
$$

### General Volumes by Slicing:

- **Given:** Base and shape of Cross-sections
- **Formula:**
  - For volumes of revolution laying on the axis with slices perpendicular to the axis
  - If slices are vertical:
    $$
    V = \int_a^b \pi[R(x)]^2 dx
    $$
  - If slices are horizontal:
    $$
    V = \int_c^d \pi[R(y)]^2 dy
    $$

### Disk Method:

- For volumes of revolution laying on the axis with slices perpendicular to the axis
- If slices are vertical:
  $$
  V = \int_a^b \pi[R(x)]^2 dx
  $$
- If slices are horizontal:
  $$
  V = \int_c^d \pi[R(y)]^2 dy
  $$

### Washer Method:

- For volumes of revolution not laying on the axis with slices perpendicular to the axis
- If slices are vertical:
  $$
  V = \int_a^b \pi[R(x)]^2 - \pi[r(x)]^2 dx
  $$
- If slices are horizontal:
  $$
  V = \int_c^d \pi[R(y)]^2 - \pi[r(y)]^2 dy
  $$

### Shell Method:

- For volumes of revolution with slices parallel to the axis
- If slices are vertical:
  $$
  V = \int_a^b 2\pi rh dx
  $$
- If slices are horizontal:
  $$
  V = \int_c^d 2\pi rh dy
  $$

### Physical Applications:

<table>
<thead>
<tr>
<th><strong>Physics Formulas</strong></th>
<th><strong>Associated Calculus Problems</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mass:</strong></td>
<td>Mass of a one-dimensional object with variable linear density:</td>
</tr>
<tr>
<td>Mass = Density * Volume (for 3-D objects)</td>
<td>Mass = \int_a^b (linear density) \frac{dx}{distance} = \int_a^b \rho(x)dx</td>
</tr>
<tr>
<td>Mass = Density * Area (for 2-D objects)</td>
<td></td>
</tr>
<tr>
<td>Mass = Density * Length (for 1-D objects)</td>
<td></td>
</tr>
<tr>
<td><strong>Work:</strong></td>
<td>Work to stretch or compress a spring (force varies):</td>
</tr>
<tr>
<td>Work = Force * Distance</td>
<td>Work = \int_a^b (force)dx = \int_a^b F(x)dx = \int_a^b kx \frac{dx}{distance}</td>
</tr>
<tr>
<td>Work = Mass * Gravity * Distance</td>
<td></td>
</tr>
<tr>
<td>Work = Volume * Density * Gravity * Distance</td>
<td></td>
</tr>
<tr>
<td><strong>Force/Pressure:</strong></td>
<td>Force of water pressure on a vertical surface:</td>
</tr>
<tr>
<td>Force = Pressure * Area</td>
<td>Force = \int_c^d (gravity)(density)(depth) \frac{dy}{area}</td>
</tr>
<tr>
<td>Pressure = Density * Gravity * Depth</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Force = \int_c^d 9.8 \rho \ast (a - y) \ast w(y)dy (in metric)</td>
</tr>
</tbody>
</table>
Integration by Parts:

Knowing which function to call \( u \) and which to call \( dv \) takes some practice. Here is a general guide:

\[
\begin{align*}
\text{u} & \quad \text{Inverse Trig Function} & (\sin^{-1}x, \arccos x, \text{etc}) \\
\text{Logarithmic Functions} & (\log 3x, \ln(x+1), \text{etc}) \\
\text{Algebraic Functions} & (x^3, x + 5, 1/x, \text{etc}) \\
\text{Trig Functions} & (\sin(5x), \tan(x), \text{etc}) \\
\text{Exponential Functions} & (e^{3x}, 5^{3x}, \text{etc})
\end{align*}
\]

Functions that appear at the top of the list are more likely to be \( u \), functions at the bottom of the list are more likely to be \( dv \).

Trig Integrals:

<table>
<thead>
<tr>
<th>Integrals involving ( \sin(x) ) and ( \cos(x) ):</th>
<th>Integrals involving ( \sec(x) ) and ( \tan(x) ):</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. If the power of the sine is odd and positive:</td>
<td>1. If the power of ( \sec(x) ) is even and positive:</td>
</tr>
<tr>
<td>\textbf{Goal:} ( u = \cos x )</td>
<td>\textbf{Goal:} ( u = \tan x )</td>
</tr>
<tr>
<td>i. Save a ( du = \sin(x)dx )</td>
<td>i. Save a ( du = \sec^2(x)dx )</td>
</tr>
<tr>
<td>ii. Convert the remaining factors to ( \cos(x) )</td>
<td>ii. Convert the remaining factors to ( \tan(x) )</td>
</tr>
<tr>
<td>(using ( \sin^2 x = 1 - \cos^2 x ).)</td>
<td>(using ( \sec^2 x = 1 + \tan^2 x ).)</td>
</tr>
<tr>
<td>2. If the power of the cosine is odd and positive:</td>
<td>2. If the power of ( \tan(x) ) is odd and positive:</td>
</tr>
<tr>
<td>\textbf{Goal:} ( u = \sin x )</td>
<td>\textbf{Goal:} ( u = \sec(x) )</td>
</tr>
<tr>
<td>i. Save a ( du = \cos(x)dx )</td>
<td>i. Save a ( du = \sec(x) \tan(x)dx )</td>
</tr>
<tr>
<td>ii. Convert the remaining factors to ( \sin(x) )</td>
<td>ii. Convert the remaining factors to ( \sec(x) )</td>
</tr>
<tr>
<td>(using ( \cos^2 x = 1 - \sin^2 x ).)</td>
<td>(using ( \sec^2(x) - 1 = \tan^2(x) ).)</td>
</tr>
<tr>
<td>3. If both ( \sin(x) ) and ( \cos(x) ) have even powers:</td>
<td></td>
</tr>
<tr>
<td>Use the half angle identities:</td>
<td>• If there are no sec(x) factors and the power of</td>
</tr>
<tr>
<td>i. ( \sin^2(x) = \frac{1}{2}(1 - \cos(2x)) )</td>
<td>( \tan(x) ) is even and positive, use ( \sec^2(x) - 1 =</td>
</tr>
<tr>
<td>ii. ( \cos^2(x) = \frac{1}{2}(1 + \cos(2x)) )</td>
<td>( \tan^2(x) ) to convert one ( \tan^2(x) ) to ( \sec^2(x) )</td>
</tr>
<tr>
<td>\textit{If nothing else works, convert everything to sines}</td>
<td>• Rules for sec(x) and tan(x) also work for csc(x) and</td>
</tr>
<tr>
<td>and cosines.</td>
<td>( \cot(x) ) with appropriate negative signs</td>
</tr>
</tbody>
</table>

Trig Substitution:

<table>
<thead>
<tr>
<th>Expression</th>
<th>Substitution</th>
<th>Domain</th>
<th>Simplification</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{a^2 - u^2} )</td>
<td>( u = a \sin \theta )</td>
<td>(- \frac{\pi}{2} \leq \theta \leq \frac{\pi}{2})</td>
<td>( a^2 - u^2 = a \cos \theta )</td>
</tr>
<tr>
<td>( \sqrt{a^2 + u^2} )</td>
<td>( u = a \tan \theta )</td>
<td>(- \frac{\pi}{2} &lt; \theta &lt; \frac{\pi}{2})</td>
<td>( a^2 + u^2 = a \sec \theta )</td>
</tr>
<tr>
<td>( \sqrt{u^2 - a^2} )</td>
<td>( u = a \sec \theta )</td>
<td>( 0 \leq \theta \leq \pi, \theta \neq \frac{\pi}{2} )</td>
<td>( u^2 - a^2 = a \tan \theta )</td>
</tr>
</tbody>
</table>

Partial Fractions:

<table>
<thead>
<tr>
<th>Linear factors:</th>
<th>Irreducible quadratic factors:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{P(x)}{(x - r_1)^m} = \frac{A}{(x - r_1)} + \frac{B}{(x - r_1)^2} + \ldots + \frac{Y}{(x - r_1)^{m-1}} )</td>
<td>( \frac{P(x)}{(x^2 + r_1)^m} = \frac{Ax + B}{(x^2 + r_1)} + \frac{Cx + D}{(x^2 + r_1)^2} + \ldots + \frac{Wx + X}{(x^2 + r_1)^{m-1}} )</td>
</tr>
<tr>
<td>If the fraction has multiple factors in the denominator, we just</td>
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</tr>
<tr>
<td>add the decompositions.</td>
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</tr>
</tbody>
</table>