MSLC Computing Derivatives Handout

Definition of the Derivative:

The derivative of a function f is another function f' whose value at any number a is:

$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$, provided that this limit exists.

Other Forms of the Definition of the Derivative:

$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$	$f'(x)=\lim_{t\to x}\frac{f(t)-f(x)}{t-x}$	$f'(a)=\lim_{t\to a}\frac{f(t)-f(a)}{t-a}$

Table of Key Derivatives:

Exponent and Log functions	$\frac{d}{dx}e^x=e^x$	$\frac{d}{dx}a^x=a^x\ln a$	$\frac{d}{dx}\ln x=\frac{1}{x}$
Trigonometric functions	$\frac{d}{dx}\sin x=\cos x$ $\frac{d}{dx}\csc x=-\csc x\cot x$	$\frac{d}{dx}\cos x=-\sin x$ $\frac{d}{dx}\sec x=\sec x\tan x$	$\frac{d}{dx}\tan x=\sec^2 x$ $\frac{d}{dx}\cot x=-\csc^2 x$
Inverse Trig functions	$\frac{d}{dx}\sin^{-1}x=\frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\cos^{-1}x=-\frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\tan^{-1}x=\frac{1}{1+x^2}$

Derivative Rules

- $\frac{d}{dx}c=0$ — derivative of ANY constant (anything without an x)
- $\frac{d}{dx}(c\cdot f)=c\cdot f'$ — derivative of a constant times a function
- $\frac{d}{dx}(x^n)=nx^{n-1}$ — the Power Rule
- $(f\pm g)'=f'\pm g'$ — sum or difference of functions
- $\left(f\cdot g\right)'=f'\cdot g+f\cdot g'$ — the Product Rule
- $\left(\frac{f}{g}\right)'=\frac{f'\cdot g-f\cdot g'}{g^2}$ — the Quotient Rule
- $[f(g(x))]'=f'(g(x))\cdot g'(x)$ — the Chain Rule

Implicit Differentiation

If we want to find $\frac{dy}{dx}$, we think of y as implicitly defined as a function of x.

- When we differentiate x, we get 1.
- When we differentiate y, we get $\frac{dy}{dx}$ or y' (either is fine).
- Then we solve for $\frac{dy}{dx}$.

Logarithmic Differentiation

Used when the function is complicated or for functions with an x in base and in the exponent.

Option 1: Take the log of both sides, simplify with log properties, differentiate (implicit chain rule on y will always happen on the left side), then solve for y'.

Ex. $y=x^x \Rightarrow \ln y=\ln x^x=x\ln x \Rightarrow \frac{d}{dx}\left(\ln y\right)=\frac{d}{dx}\left(x\ln x\right) \Rightarrow$

$\frac{1}{y}y'=1*\ln x+x*\frac{1}{x}=\ln x+1 \qquad \Rightarrow \qquad y'=y(\ln x+1)=x^x(\ln x+1)$

Option 2: Take e^ln(your equation), simplify with log properties, differentiate (not implicit).

Ex. $y=x^x \Rightarrow y=e^{\ln x^x}=e^{x\ln x} \Rightarrow \frac{dy}{dx}=\frac{d}{dx}\left(e^{x\ln x}\right) \Rightarrow$

$y'=e^{x\ln x}\left(1*\ln x+x*\frac{1}{x}\right)=e^{x\ln x}\left(\ln x+1\right)$