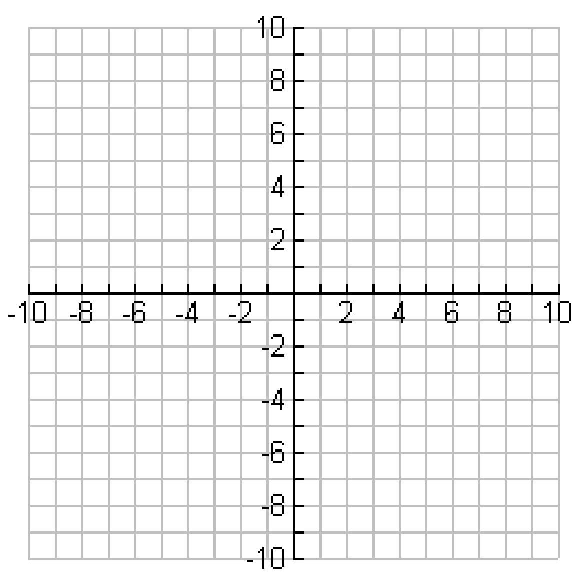
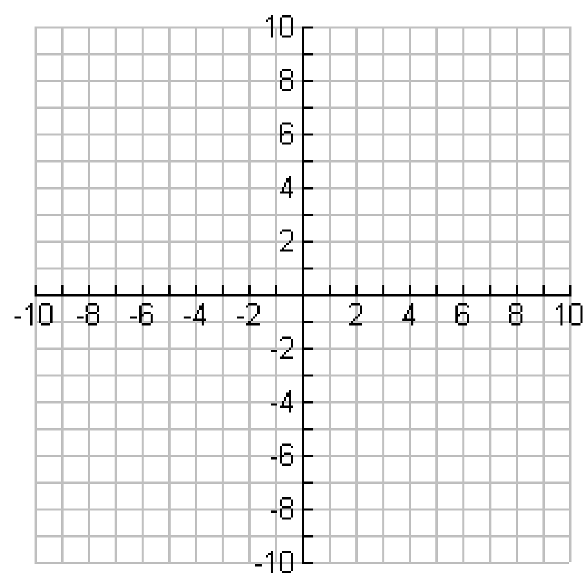
MSLC Workshop Series
Math 1151
Limits and Continuity

Warm-up: Let $f(x) = \frac{x^2 - 6x + 8}{x - 2}$ and let $g(x) = x - 4$. Are *f* and *g* equivalent functions? Why or why not?

Draw the graphs of $y = f(x)$ and $y = g(x)$.



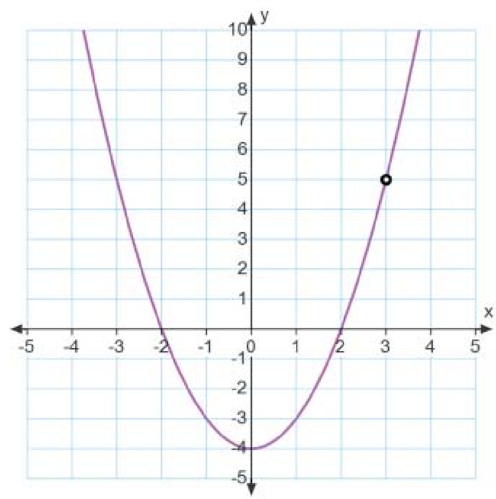


Limits:

There is a technical definition, but for us, the intuitive definition will do just fine. Here it is:

The limit *L* is the value the function "gets close to" if we make the *x* values "get close to" (but not equal to)

Quick Check: Find the value of the $\lim_{x \to 3} f(x)$ from the graph.



Continuity:

Def A function $f(x)$ is continuous at $x = a$ if and only if $\lim_{x \to a} f(x) = f(a)$.

This definition actually states three things:

1.

2.

3.

Part 1: Limits and Continuity given by a Graph

Let $y = f(x)$ be given by the graph below. Use the graph to find the following.

1. a. $f(1) =$

   b. $\lim_{x \to 1} f(x) =$

   c. Is f continuous at $x = 1$?

2. a. $\lim_{x \to -2^-} f(x) =$

   b. $\lim_{x \to -2^+} f(x) =$

   c. $\lim_{x \to -2} f(x) =$

   d. Is f continuous at $x = -2$?

3. a. $\lim_{x \to -5^-} f(x) =$

   b. $\lim_{x \to -5^+} f(x) =$

   c. $\lim_{x \to -5} f(x) =$

   d. $f(-5) =$

   e. Is f continuous at $x = -5$?

4. a. $\lim_{x \to 3^-} f(x) =$

   b. $\lim_{x \to 3^+} f(x) =$

   c. $\lim_{x \to 3} f(x) =$

   d. $f(3) =$

   e. Is f continuous at $x = 3$?

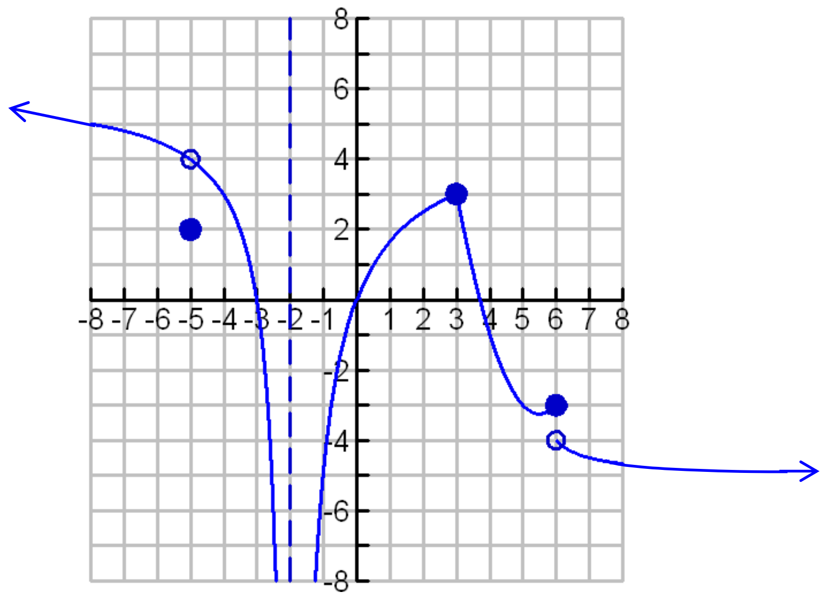
5. a. $\lim_{x \to 6^-} f(x) =$

   b. $\lim_{x \to 6^+} f(x) =$

   c. $\lim_{x \to 6} f(x) =$

   d. $f(6) =$

   e. Is f continuous at $x = 6$?



Part 2: Limits, Continuity, and Differentiability given by an Equation

Hints about finding Limits:

- Plugging in nearby numbers will get you no credit unless the directions specifically say to do so.
- For any function which is CONTINUOUS, you can find the limit just by plugging in the number as long as the answer is defined.
  - In particular we know the following functions and all their combinations are continuous wherever they are defined:

    ______________________________ ______________________________

    ______________________________ ______________________________

    ______________________________ ______________________________

    ____

  - Ways you can combine continuous functions to get another continuous function:

    ______________________________ ______________________________

    ______________________________ ______________________________

    ______________________________ ______________________________

- What do you do if the function is undefined?

$\dfrac{\text{zero}}{\text{nonzero number}}$ $\dfrac{\text{nonzero number}}{\text{zero}}$ $\dfrac{\text{zero}}{\text{zero}}$

$\dfrac{\text{number}}{\text{infinity}}$ $\dfrac{\text{infinity}}{\text{infinity}}$

Limit Examples:

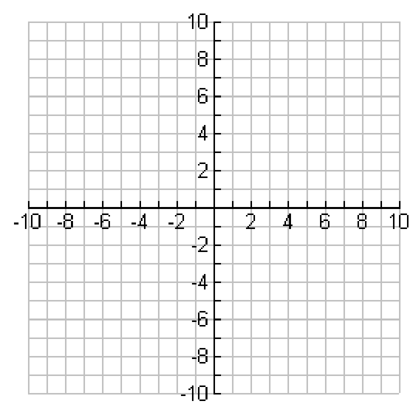
1. $\lim_{x\to 4}\dfrac{3-2x}{(x-4)(x+2)}$

2. $\lim_{x\to 4}\dfrac{x^2-4x}{x^2-3x-4}$

3. $\lim_{h\to 0}\dfrac{\sqrt{1+h}-1}{h}$

4. $f(x)=\begin{cases}4-x^2 & x\le 2\\ x-1 & x>2\end{cases}$

   a. Find $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^+} f(x)$.
   b. Does $\lim_{x\to 2} f(x)$ exist?
   c. Sketch a graph of f(x).



5. Challenge Problem! $\lim_{x\to -\infty}\dfrac{3x^3}{\sqrt{9x^6+x}}$

Continuity Examples: Determine if the following functions are continuous at the given x-value.

1. $f(x)=\begin{cases} 3x^2+7 & x<2 \\ \sin(x) & x>2 \end{cases}$, x=2

2. $g(x)=\begin{cases} \frac{x^2-2x-8}{x-4} & x\neq 4 \\ 7 & x=4 \end{cases}$, x=4

Comparison Chart of Limits vs. Continuity

	Limits	*Continuity*
Conceptually	Where is the function headed (y-value) as you get near a certain x-value?	Can you draw it without picking up your pencil?
Graphically	No jumps or infinite squiggles, ignore the point itself	No holes, breaks, or infinite squiggles
Algebraically	1) Limits from both sides have to agree	1) Limits from both sides have to agree 2) The y-value of the point has to agree with the limit
Math Notation ** And fine print*	1) $\lim_{x\to a^+} f(x)=\lim_{x\to a^-} f(x)$ *f(x) is defined on an interval on both sides of a	1) $\lim_{x\to a^+} f(x)=\lim_{x\to a^-} f(x)$ 2) $f(a)$ is defined and $f(a)=\lim_{x\to a} f(x)$