Math 1131 Review for Midterm 3 (13.6-14.10)

1. Evaluate
   a. \( \int \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 \, dx \)
   b. \( \int x^3 \sqrt{x^4 + 5} \, dx \)
   c. \( \int \frac{\ln(xe^x)}{x} \, dx \)
   d. \( \int_{0}^{1} \frac{2x^2 + 11x + 5}{2x + 1} \, dx \)
   e. \( \int_{1}^{2} \frac{e^{\ln(x)}}{x} \, dx \)
   f. \( \int \left( \frac{5}{3x + 7} + \frac{3}{\sqrt{x}} \right) \, dx \)

2. In the problem below \( \frac{dr}{dq} \) is a marginal revenue function. Find the demand function.
   \( \frac{dr}{dq} = 7500 - 7(2q + 4q^3) \)

3. Find \( y \), subject to the given conditions. \( y'' = -3x^2 + 2x + 7; \quad y'(1) = 7; \quad y(1) = 0. \)

4. Find an approximate area \( S_3 \) of the region bounded by the given curves in the first quadrant. (Use the right-hand endpoint of each subinterval.) \( f(x) = x^3; \quad y = 0; \quad x = 6 \)

5. A company manufactures and sells \( x \) mp3 players per week. If the weekly cost and demand equations are given by: \( C(x) = 8000 + 5x \) and \( p = 14 - \frac{x}{4000}, \quad 0 \leq x \leq 25,000 \)
   Find the production level that maximizes profit.

6. A company wishes to manufacture a box with a volume of 36 cubic feet that is open on top and is twice as long as it is wide. Find the width of the box that can be produced using the minimum amount of material.

7. A real estate firm owns 100 apartments. At $400 per month, each apartment can be rented. However, for each $10-per-month increase, there will be two vacancies with no possibility of filling them. What rent per apartment will maximize the monthly revenue?

8. A manufacturer’s marginal-cost function is \( \frac{dc}{dq} = 0.004q^2 - 0.5q + 50. \) If \( c \) is in dollars, determine the cost involved to increase production from 90 to 180 units.

9. Find the area in the region bounded by \( y = x^3 \) and \( y = x \).

10. Find the area of the region bounded by \( y = x + 4 \) and \( y = x^2 - 2 \).

11. Find the area of the region bounded by \( y = \sqrt{x}, \ y = -x + 6, \) and the x-axis. Integrate in terms of \( x \).

12. The demand for a product is given by \( p = 500 - q^2 \) and its supply is given by \( p = 30q + 100 \). Find the consumers’ surplus under market equilibrium.
Answers to 1131 Review for Midterm 3

1. a. \( \frac{x^2}{2} + \ln|x| + 2x + C \)
   
b. \( \frac{1}{6} (x^4 + 5)^{\frac{3}{2}} + C \)
   
c. \( \frac{(\ln(x))^2}{2} + x + C \)
   
d. \( 5\frac{1}{2} \)
   
e. \( 2e^{\sqrt{x}} - 2e \)
   
f. \( \frac{5}{3} \ln(3x + 7) + \frac{3}{2} x^{\frac{2}{3}} + C \)

2. \( p = 7500 - 7(q + q^3) \)

3. \( \frac{x^4}{4} + \frac{x^3}{3} + \frac{7x^2}{2} - \frac{43}{12} \)

4. 112

5. 18,000 mp3 players per week

6. 3 feet

7. $450

8. $5229

9. 1.5

10. \( \frac{125}{6} \)

11. \( \frac{22}{3} \)

12. \( \frac{2000}{3} = 666.67 \)