Math 1131 Review for Midterm 3

1. Evaluate
   a. $\int \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 \, dx$
   b. $\int x^3 \sqrt{x^4 + 5} \, dx$
   c. $\int \frac{\ln(xe^x)}{x} \, dx$
   d. $\int_0^1 \frac{2x^2 + 11x + 5}{2x + 1} \, dx$
   e. $\int_1^2 \frac{e^x}{\sqrt{x}} \, dx$
   f. $\int_1^2 \left( \frac{5}{3x+7} + \frac{3}{x^2} \right) \, dx$

2. In the problem below $\frac{dr}{dq}$ is a marginal revenue function. Find the demand function.
   \[ \frac{dr}{dq} = 7500 - 7(2q + 4q^3) \]

3. Find $y$, subject to the given conditions. $y'' = -3x^2 + 2x + 7; \quad y'(1) = 7; \quad y(1) = 0.$

4. Find an approximate area $S_3$ of the region bounded by the given curves in the first quadrant. (Use the right-hand endpoint of each subinterval.) $f(x) = x^2; \quad y = 0; \quad x = 6$

5. A manufacturer's marginal-cost function is $\frac{dc}{dq} = 0.004q^2 - 0.5q + 50$. If $c$ is in dollars, determine the cost involved to increase production from 90 to 180 units.

6. Find the area in the region bounded by $y = x^3$ and $y = x$.

7. Find the area of the region bounded by $y = x + 4$ and $y = x^2 - 2$.

8. Find the area of the region bounded by $y = \sqrt{x}, \quad y = -x + 6$, and the $x$-axis. Integrate in terms of $x$.

9. The demand for a product is given by $p = 500 - q^2$ and its supply is given by $p = 30q + 100$. Find the consumers’ surplus under market equilibrium.
Answers to 1131 Review for Midterm 3

1. a. $\frac{x^2}{2} + \ln|x| + 2x + C$

    b. $\frac{1}{6}(x^4 + 5)^{\frac{3}{2}} + C$

    c. $\frac{(\ln(x))^2}{2} + x + C$

    d. $5\frac{1}{2}$

    e. $2e^{x^2} - 2e$

    f. $\frac{x}{2}\ln(3x + 7) + \frac{3}{5}x^{\frac{3}{2}} + C$

2. $p = 7500 - 7(q + q^3)$

3. \(-\frac{x^4}{4} + \frac{x^3}{3} + \frac{7x^2}{2} - \frac{43}{12}\)

4. 112

5. $5229$

6. 1.5

7. $\frac{125}{6}$

8. $\frac{22}{3}$

9. $\frac{2000}{3} \approx 666.67$