

1. Consider the function $y = f(x)$ as defined by the graph below.

a) Evaluate the following:

i) $\lim_{x \rightarrow -\infty} f(x)$ ii) $\lim_{x \rightarrow -6} f(x)$ iii) $\lim_{x \rightarrow -2} f(x)$

iv) $\lim_{x \rightarrow 3} f(x)$ v) $\lim_{x \rightarrow 6} f(x)$ vi) $\lim_{x \rightarrow \infty} f(x)$

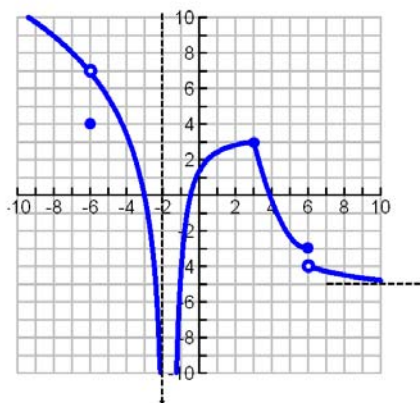
b) What is the domain of $f(x)$ in interval notation?
(Assume the function goes past the visible portion.)
What is the range of $f(x)$?

c) Where is $f(x)$ discontinuous? Where is $f(x)$ continuous?

d) Where is $f(x)$ not differentiable?

e) Estimate $f'(1)$ and give the equation of the tangent line there.

f) Estimate $f'(-3)$ and give the equation of the tangent line there.



2. Find the following limits or state that the limit does not exist. (Do not use your calculator to help with these!)

a) $\lim_{z \rightarrow 2} \frac{z^2 - 4}{z^2 + z - 6}$

b) $\lim_{x \rightarrow \infty} e^{-x} \sqrt{x}$

c) $\lim_{x \rightarrow 0} \frac{x^2 + 6x}{\sin x}$

d) $\lim_{t \rightarrow 0} \frac{\sin^2(3t)}{t^2 - t}$

e) $\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{\sin x \cos x}$

f) $\lim_{u \rightarrow -\infty} \frac{2u}{\sqrt{u^2 + 3u}}$

3. Let $f(x) = \frac{x+2}{2x^2+1}$. Use the definition of the derivative to calculate $f'(1)$. What is the equation of the tangent line there?

4. Differentiate the following using derivative rules.

a) $f(x) = \sin^3 x - \sin(x^3)$ b) $g(x) = \sqrt[4]{(x^2 - 1) e^{5x^2 + \ln 16}}$ c) $h(x) = (x^2 - 2x)^{x^3 + 5x + 4}$

5. Let $z(x) = \left(\frac{(x+1)^2 \cos x}{x(\csc x)} \right)^3$. Find $z'(x)$ with logarithmic differentiation.

6. Suppose $x^3 y - xy^2 = 0$. Find the equations of the tangent lines at the point (1,1).

7. Use implicit differentiation to find y' if $xy - \cos(xy) + y^2 = 3$.

8. Find the point (x, y) on the graph $y = \sqrt{x}$ nearest the point (4, 0).

9. Let $f(x) = \begin{cases} ax^2 + bx + 1 & \text{on } [0, 2] \\ x - 1 & \text{on } (2, \infty) \end{cases}$ Find a and b so that $f(x)$ is continuous and differentiable everywhere.

Final Exam Review

10. Determine the horizontal (or slant) and vertical asymptotes of

$$\text{a) } t(x) = \frac{3x-1}{x+1}$$

$$\text{b) } z(x) = \frac{x^2 + 4x + 3}{x + 2}$$

11. Sketch the graph of $2x^3 - 9x^2 + 20$ using your knowledge of the information obtained from the original function and the first and second derivatives. Be sure to indicate all important features. (Includes increasing, decreasing, max, min, concavity, intercepts, etc)

12. Sketch a graph of $f(x)$ with the following properties:

- ◆ $f(-7) = f(-4) = f(1) = f(6) = 0$, $f(-2) = 4$, $f(4) = -2$, $f'(-2) = f'(4) = f''(0) = 0$
- ◆ $\lim_{x \rightarrow -\infty} f(x) = -2$, $\lim_{x \rightarrow \infty} f(x) = 2$, $\lim_{x \rightarrow -5^-} f(x) = \infty$, $\lim_{x \rightarrow -5^+} f(x) = -\infty$
- ◆ $f'(x) > 0$ on $(-\infty, -5) \cup (-5, -2) \cup (4, \infty)$, $f'(x) < 0$ on $(-2, 4)$
- ◆ $f''(x) > 0$ on $(-\infty, -5) \cup (0, 6)$, $f''(x) < 0$ on $(-5, 0) \cup (6, \infty)$

13. Find the most general antiderivatives of the following functions.

$$\text{a) } f(x) = x^8 - \frac{2}{x^e} - 17$$

$$\text{b) } f(x) = 4 \csc^2 x + \frac{1}{x}$$

14. Find the particular antiderivative of the following.

$$\text{a) } f(x) = e^x + x, F(0) = 2 \quad \text{b) } g(x) = \frac{12x}{(3x^5)^3}, G(1) = 3$$

15. Find Δy and dy if $y = \frac{\sqrt{x}}{x+1}$, $x = 4$, $\Delta x = .1$. Then, use differentials to estimate $y(4.1)$.

16. a) Express the following sum in the form $\sum_{i=1}^n a_i$.

$$\left[\left(2 + \frac{5}{n} \right)^3 + 1 \right] \left(\frac{5}{n} \right) + \left[\left(2 + \frac{10}{n} \right)^3 + 1 \right] \left(\frac{5}{n} \right) + \left[\left(2 + \frac{15}{n} \right)^3 + 1 \right] \left(\frac{5}{n} \right) + \cdots + \left[\left(2 + \frac{5n}{n} \right)^3 + 1 \right] \left(\frac{5}{n} \right)$$

b) The above is a right Riemann Sum for the definite integral $\int_2^b f(x) dx$. Find b and $f(x)$.

c) Find the value of the above sum as $n \rightarrow \infty$.

d) The above sum can also be a right Riemann Sum for the definite integral $\int_0^b f(x) dx$. Find the new function and bounds and compute the value of the sum as $n \rightarrow \infty$.

17. Express the definite integral $\int_1^3 x^2 dx$ as a right Riemann Sum and compute the sum using sigma algebra and the limit as $n \rightarrow \infty$. Verify that the sum you compute is the same as computing the integral using the Fundamental Theorem of Calculus.

18. Suppose $\int_0^5 f(x) = 10$, $\int_2^0 f(x) = 3$, $\int_3^5 f(x) = 6$. Then $\int_2^3 f(x) = ?$

19. Find the following integrals:

a. $\int \frac{\sec^2 x}{\tan^6 x} dx$

b. $\int (\cos(2\theta) - 2\sin \theta) d\theta$

c. $\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx$

d. $\int (x+1)e^{(x+1)^2} dx$

e. $\int \frac{2}{x(\ln x^2)^2} dx$

f. $\int_{-1}^2 (3x^2 + x) dx$

g. $\int_1^5 \sqrt{2x-1} dx$

h. $\int_0^1 x^3 (4-5x^4)^6 dx$

i. $\int_{-2}^2 |x-1| dx$

20. a. Let $g(x) = \int_1^x \cot^5(\sqrt{t}) dt$. Find $g'(x)$.

b. Let $f(x) = \int_{x^3}^{x^2} e^{t^2} \sin(2t) dt$. Find $f'(x)$.

21. Consider the integral $\int_0^x f(t) dt = \cos^2(2x) + A$. Determine $f(t)$ and the constant A .

22. A boat is being pulled toward a dock by means of a rope attached to the front tip of the bow. Initially there are 30 feet of rope out and the rope is taught and being reeled in by a circular device the top of which is 10 feet higher than the point where the rope is attached to the boat. This circular device has a radius of 1 foot and turns at the rate of one revolution every pi seconds. How fast is the boat moving along the water when there are 15 feet of rope out?

23. A baseball diamond is a square which is 90 feet on each side, and the pitcher's mound is at the center of the square. If a pitcher throws a baseball at 93 miles per hour, how fast is the distance between the ball and first base changing as the ball crosses home plate.

24. Consider a rectangle of perimeter 12 inches. Form a cylinder by revolving this rectangle about one of its edges. What dimensions of the rectangle will result in a cylinder of maximum volume?

25. Suppose a carpenter has 20 feet of window trim. The plans call for a window to be built such that the window is rectangular with an equilateral triangle at the top. Find the dimensions of the window such that the area of the window is maximized. What is the area of the window?

26. Suppose that a function $T(t)$ is a differentiable function that models how the temperature changes over one 24-hour period. Let $t=0$ at midnight, with t measured in hours. Refer to the following table:

| | | | | | | | | | | | | | |
|---|----|----|----|----|----|----|----|----|----|----|----|----|----|
| t | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| T | 50 | 47 | 43 | 42 | 55 | 64 | 73 | 77 | 83 | 82 | 75 | 65 | 58 |

- Is there a time between 6am and noon when the temperature is 61 degrees?
- Find the average rate of change of the temperature from 6am to noon. Is there a time between 6am and noon when the temperature is changing at this rate?