1. A farmer needs to irrigate his field. He has water stored in a tank that has the shape of: $y = x^2$, bounded by x=0, y=0, y=25 and rotated about the y-axis. The water must be pumped out through the top of the tank and into the field. How much work is done pumping the water out of the tank for use in the field?
   (Note: all measurements are in meters, kilograms and seconds).

2. A rope is used to pull a 2 lb weight up a 50 ft tall wall. The rope weighs 0.5 lb/ft. What work is done in lifting the rope and weight to the top of the wall?

3. Find the volumes of the following solid bounded by the x-axis, $y = \sqrt{x^2 - 1}$ and x=10 with semicircular cross-sections $\perp$ to the x-axis.

4. Set up the integral to find the volume of the solid bounded by the functions $y = (x-1)^2$, $y = (x-5)^2$ and y=0
   a) rotated about the line x=10
   b) rotated about y=4

5. Find the total area enclosed between the curves $x = \sin y$ and $y = \frac{\pi}{2}x$. Note that the graphs intersect when $y = \pm\frac{\pi}{2}$ and when $y = 0$.

6. Find the length of the curve $x = \frac{1}{3}\sqrt{y}(y-3),\ 1 \le y \le 9$.

7. Find the following integrals:

a) $\int_0^2 x^2 \sin(3x)dx$ b) $\int x^3 \sin\left(x^2\right)dx$ c) $\int_{-\infty}^{\infty} xe^{-x^2}dx$ d) $\int_3^{\infty} \frac{2x+1}{(x-2)(x-1)^2}dx$

f) $\int_0^2 \frac{x+1}{x^2+4}dx$ g) $\int \ln(1+x^2)dx$ h) $\int_0^{1/2} \frac{3}{\sqrt{4-16x^2}}dx$ i) $\int \sin^3(x)\cos^4(x)dx$

j) $\int_0^1 \arcsin x dx$ k) $\int_1^8 \frac{4}{x^{4/3}+2x-3x^{2/3}}dx$

8. Determine whether the sequence is convergent or divergent. If it is convergent, find its limit.

a) $a_n = \left(1+\frac{1}{7n}\right)^n$ b) $a_n = \frac{2^{n+3}}{5^n}$ c) $a_n = \frac{\sqrt{n+1}-\sqrt{n-1}}{n}$ d) $a_n = \frac{n^3+4}{\sqrt{5n^7+4}}$

9. Find the sum of the following series: $\sum_{n=1}^{\infty} \frac{2^{3n}}{11^n}$

10. Find the interval of convergence of the follow power series. (Do not worry about the end points of the interval!)

a) $\sum_{n=1}^{\infty} \frac{n(-5)^n x^n}{5n+1}$ b) $\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{(n+1)2^n}$

11. Approximate $\sin\left(\frac{\pi}{5}\right)$ using the first four non-zero terms of the Taylor series about $a = \frac{\pi}{6}$. Estimate the error of this approximation.

12. Find the Taylor Series about a for the following functions:
   a) $f(x) = x^6 e^{3x},\ a = 0$
   b) $f(x) = \dfrac{d}{dx}\left(x^6 e^{3x}\right),\ a = 0$
   c) $f(x) = (x-6)^2,\ a = 0$
   d) $f(x) = (x-6)^2,\ a = 6$

13. Consider the parametric curve $x = \cos t, y = 8\sin t$
   a) Determine $\dfrac{dy}{dx}$ in terms of *t* and evaluate it at $t = \frac{\pi}{3}$
   b) Make a sketch of the curve showing the tangent line at the point corresponding to the given value of t.

14. Consider the parametric curve $x = 1 + 3t^2, y = 4 + 2t^3, 0 \le t \le 1$. Find the length of the curve.

15. Consider the parametric curve $x = \ln t, y = \sqrt{t}, t \ge 1$. Sketch the curve and find its regular, Cartesian equation.

16. Consider the curves $r = 1 - \sin\theta$ and $r = 1 + \cos\theta$.
   a) Graph both curves.
   b) Find all intersection points.
   c) Find the area of the region inside the cartioid $r = 1 + \cos\theta$ and outside the cardioid $r = 1 - \sin\theta$.

17. Find the Cartesian equation of polar equation $r - 3\cos\theta = 0$.

18. Consider the two vectors $\mathbf{u} = \langle 2,3,1 \rangle$ and $\mathbf{v} = \langle 2,4,-5 \rangle$.
   a) Find the angle between the vectors.
   b) Find the $\text{proj}_{\mathbf{v}}\mathbf{u}$
   c) Express **u** as the sum $\mathbf{u} = \mathbf{p} + \mathbf{n}$ where **p** is parallel to **v** and **n** is orthogonal to **v.**

19. Consider the pair of lines $\mathbf{r}(t) = \langle 4t, 1+2t, 3t \rangle$ and $\mathbf{R}(s) = \langle -1+s, -7+2s, -12+3s \rangle$.
   a) Find the point of intersection.
   b) Determine the equation of the line that is perpendicular to these two lines and passes through the point of intersection.
   c) Determine the equation of the line that is parallel to $\mathbf{R}(s)$ and passes through (0,0,0).

20. Reparameterize the curve $\mathbf{R}(t) = 2t\,\mathbf{i} + (1-3t)\,\mathbf{j}$ with respect to arc length, s, measured from the point where *t* = 0 in the direction of increasing *t*.

21. Consider the curve $\mathbf{r}(t) = t\,\mathbf{i} + \cos t\mathbf{j} + +\sin t\mathbf{k},\ 0 \le t \le 4\pi$. Find the tangent line to $\mathbf{r}(t)$ at the point $(\pi, -1, 0)$.

22. If the acceleration of a particle is $\mathbf{a}(t) = -9.8\mathbf{k}$ and the particle starts at the point (1,2,3) with the initial velocity $\mathbf{v}(t) = \mathbf{i} + 2\mathbf{k}$ find $\mathbf{r}(t)$ and the speed.

23. What force is required so that a particle of mass m has a position function $\mathbf{R}(t) = t^3\mathbf{i} + t^2\mathbf{j}$?

24. a) Write an equation of the plane through the point (9,0,-2) parallel to the plane $3x-5y=1$.
    b) Find parametric equations of the line through the point (9,0,-2) perpendicular to the plane $3x-5y=1$

25. Find an equation for the plane containing the points $P=(1,2,1),\ Q=(3,2,2),$ and $R=(4,-1,-1)$

26. Find an equation of the plane that passes through the point *P* (1,2,3) and contains the line $x=3t,\ \ y=1+t,\ \ z=2-t$

27. Let $f(x,y)=\dfrac{x^2}{y}$
    a) What is the domain of *f* ? What is the range of *f* ?
    b) Sketch the level curves $f(x,y)=k$ for $k=-1,0,$ and 4

28. Let $f(x,y)=\sqrt{y^2-4x^2}$. Find the domain and sketch the level curves of *f* .

29. Let $f(x)=\begin{cases}\dfrac{y}{\sqrt{x^2+y^2}} & if\ \ (x,y)\neq 0\\ 0 & if\ (x,y)=0\end{cases}$
    a) Is *f* continuous at (0,0)? Explain!
    b) Find $f_x(0,0)$ and $f_y(0,0)$ or show that they don't exist.
    c) Is *f* differentiable at (0,0)? Explain!

30. Let $f(x,y)=xe^{\sin(x^2y)}$. Find $f_x(1,0)$.

31. Evaluate the following limit or show that it does not exist. $\displaystyle\lim_{(x,y)\to(0,0)}\frac{2xy}{x^2+3y^2}$

32. Use the Chain Rule for functions of several variables to find $\dfrac{dz}{dt}$ if $z=e^{xy}$ and $x=3\cos t, y=t+5$

33. Use the Chain Rule to find $\dfrac{\partial z}{\partial s}$ if $z=x^2\sin y, x=s^3+t^3, y=2st$.

34. Suppose that over a certain region in the plane the electrical potential V is given by $V(x,y)=7y^2-2x+xy$.
    a) Find the rate of change of the potential at $P=(3,1)$ in the direction of the vector $\mathbf{v}=-\mathbf{i}+\mathbf{j}$
    b) In which direction does V change most rapidly at P?
    c) What is the maximum rate of change of V at P?
    d) Find the equation of the level curve that contains the point P.