**Definition of the Derivative:**

The derivative of a function \( f \) is another function \( f' \) whose value at any number \( a \) is:

\[
 f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}, \quad \text{provided that this limit exists.}
\]

**Other Forms of the Definition of the Derivative:**

\[
 f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad f'(x) = \lim_{t \to x} \frac{f(t) - f(x)}{t-x} \quad f'(a) = \lim_{t \to a} \frac{f(t) - f(a)}{t-a}
\]

**Table of Key Derivatives:**

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<th>Derivative</th>
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<td>( \frac{d}{dx} e^x = e^x ) \quad ( \frac{d}{dx} a^x = a^x \ln a ) \quad ( \frac{d}{dx} \ln x = \frac{1}{x} )</td>
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<td>Trigonometric functions</td>
<td>( \frac{d}{dx} \sin x = \cos x ) \quad ( \frac{d}{dx} \cos x = -\sin x ) \quad ( \frac{d}{dx} \tan x = \sec^2 x ) \quad ( \frac{d}{dx} \csc x = -\cot x \csc x ) \quad ( \frac{d}{dx} \sec x = \sec x \tan x ) \quad ( \frac{d}{dx} \cot x = -\csc^2 x )</td>
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<td>Inverse Trig functions</td>
<td>( \frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}} ) \quad ( \frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1-x^2}} ) \quad ( \frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2} )</td>
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**Derivative Rules**

- \( \frac{d}{dx} c = 0 \) \quad derivative of ANY constant (anything without an x)
- \( \frac{d}{dx} (c \cdot f) = c \cdot f' \) \quad derivative of a constant times a function
- \( \frac{d}{dx} (x^n) = nx^{n-1} \) \quad the Power Rule
- \( (f \pm g)' = f' \pm g' \) \quad sum or difference of functions
- \( (f \cdot g)' = f' \cdot g + f \cdot g' \) \quad the Product Rule
- \( \left( \frac{f}{g} \right)' = \frac{f' \cdot g - f \cdot g'}{g^2} \) \quad the Quotient Rule
- \( \left[ f(g(x)) \right]' = f'(g(x)) \cdot g'(x) \) \quad the Chain Rule

**Implicit Differentiation**

If we want to find \( \frac{dy}{dx} \), we think of \( y \) as implicitly defined as a function of \( x \).

- When we differentiate \( x \), we get 1.
- When we differentiate \( y \), we get \( \frac{dy}{dx} \) or \( y' \) (either is fine).
- Then we solve for \( \frac{dy}{dx} \).

**Logarithmic Differentiation**

Used when the function is complicated or for functions with an \( x \) in base and in the exponent.

**Option 1:** Take the log of both sides, simplify with log properties, differentiate (implicit chain rule on \( y \) will always happen on the left side), then solve for \( y' \).

Ex. \( y = x^x \Rightarrow \ln y = \ln x^x = x \ln x \Rightarrow \frac{d}{dx} (\ln y) = \frac{d}{dx} (x \ln x) \Rightarrow \)

\[
\frac{1}{y} \cdot y' = 1 \cdot \ln x + x \cdot \frac{1}{x} \Rightarrow \ln x + 1 \Rightarrow y' = y(\ln x + 1) = x^x (\ln x + 1)
\]

**Option 2:** Take \( e^{\ln(your\ equation)} \), simplify with log properties, differentiate (not implicit).

Ex. \( y = x^x \Rightarrow y = e^{\ln x^x} = e^{\ln x} \Rightarrow \frac{dy}{dx} = \frac{d}{dx} (e^{\ln x}) \Rightarrow y' = e^{\ln x} (1 \ln x + x \cdot \frac{1}{x}) = e^{\ln x} (\ln x + 1)
\]