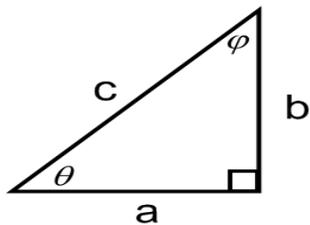


MSLC Precalculus Workshop (Math 1149 & 1150): Inverse Trig Functions

Angles versus Ratios of Numbers

In any use of trigonometry it is important to remember that there is a difference between the ratio of sides of the triangle and the reference angle, but when using inverse trig that difference is absolutely critical.



- Regular trig functions use reference angles to give a ratio of two sides of the triangle.
(Ex: $\cos \theta = \frac{a}{c}$)
- Inverse trig functions use a ratio of two sides of the triangle to give the reference angle.
(Ex: $\tan^{-1}\left(\frac{a}{b}\right) = \varphi$)

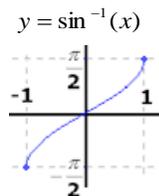
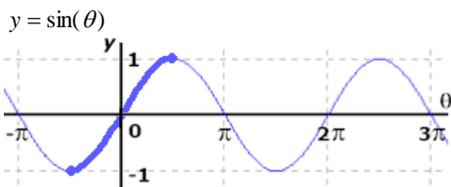
***NOTE:** You shouldn't have a ratio of two numbers in a trig function, or an angle in an inverse trig function.

Domains and Ranges

All trigonometric functions are periodic (i.e. they repeat after a certain interval).

- The end result of this is trig functions aren't one-to-one.
- This causes problems for the inverse functions. The domain has to be restricted in order for the inverse to be a function.

$$y = \sin^{-1} x \quad (= \arcsin x)$$



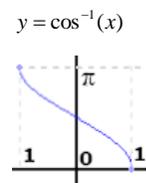
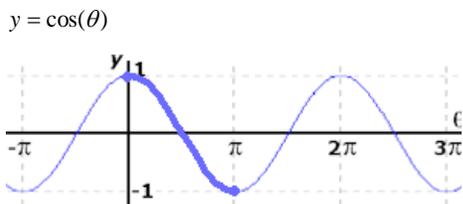
DOMAIN:

$$-1 \leq x \leq 1$$

RANGE:

$$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

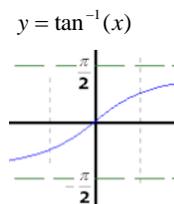
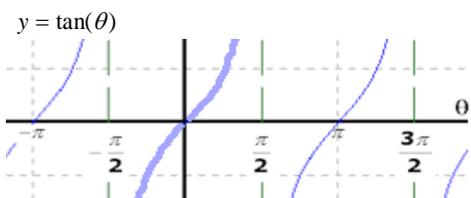
$$y = \cos^{-1} x \quad (= \arccos x)$$



$$-1 \leq x \leq 1$$

$$0 \leq y \leq \pi$$

$$y = \tan^{-1} x \quad (= \arctan x)$$



$$-\infty < x < \infty$$

$$-\frac{\pi}{2} < y < \frac{\pi}{2}$$

Boiled down, this means **inverse sine** and **inverse tangent** will give angles in **Quadrants I or IV**

(specifically, θ will be between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$), and **inverse cosine** will give angles in **Quadrants I or**

II (specifically, θ will be between 0 and π).

Simplifying an inverse trigonometric function of a specific value

Example: Simplify $\tan^{-1}(-1)$.

This is really giving you the equation $\tan^{-1}(-1) = \theta$, which means $\tan \theta = -1$.

This happens when $\theta = \frac{3\pi}{4}$ (which is in Quadrant II) or when $\theta = \frac{5\pi}{4}$ (which is in Quadrant IV).

Due to the restricted range for inverse tangent our answers have to be in Quadrant I or Quadrant IV.

This means we can't use $\theta = \frac{3\pi}{4}$ (it's in Quadrant II), so we look at $\theta = \frac{5\pi}{4}$.

Even though $\theta = \frac{5\pi}{4}$ is in Quadrant IV, there's still a problem. The angle $\frac{5\pi}{4}$ is outside the restricted interval

for the range of inverse tangent $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. We compensate for this by using an angle co-terminal to $\frac{5\pi}{4}$, but

is inside the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. That angle is $\theta = -\frac{\pi}{4}$, so $\tan^{-1}(-1) = -\frac{\pi}{4}$.

Now it's your turn. Simplify the following: (Remember that $\text{trig}^{-1}(r) = \theta$ means $\text{trig}(\theta) = r$.)

1. $\arctan(\sqrt{3})$

2. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

3. $\arccos\left(-\frac{1}{2}\right)$

4. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

There are two types of problems that involve both trigonometric and inverse trigonometric functions:

1) Inverse Trig (Trig)

Example 1: Simplify $\sin^{-1}\left(\sin\left(\frac{5\pi}{3}\right)\right)$.

Do not fall into the trap that the \sin^{-1} and the \sin will just cancel themselves out leaving you with $\frac{5\pi}{3}$. Unfortunately, this problem isn't quite that easy. The reason this has to do with the restrictions on the range of inverse sine; the angle you get for your answer has to be between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, and $\frac{5\pi}{3}$ is too big for that interval. So we'll follow the order of operations and do what's

inside the parentheses first, $\sin\left(\frac{5\pi}{3}\right)$. We know $\sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ (from the unit circle, the 30-60-90 triangle in Quadrant IV, etc.), so we're now looking for $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \theta$. This means that

$\sin(\theta) = -\frac{\sqrt{3}}{2}$ and since inverse sine has to be in either Quadrant I or IV, because we need a negative

answer for sine we must be in Quadrant IV. The angle in Quadrant IV where $\sin(\theta) = -\frac{\sqrt{3}}{2}$ and

$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ is when $\theta = -\frac{\pi}{3}$.

2) Trig (Inverse Trig)

Example 2: Simplify $\cos\left(\arccos\left(-\frac{\sqrt{3}}{2}\right)\right)$.

From our earlier work, we know that it's not a guarantee that the cosine and arccosine will just cancel themselves out, so we're going to need to do some work to get the answer to this problem.

Again we'll start with what's inside the parentheses, $\arccos\left(-\frac{\sqrt{3}}{2}\right)$, which also means that

$\cos(\theta) = -\frac{\sqrt{3}}{2}$. Our angle θ has to be in either Quadrant I or II as a result of the restrictions on the range of inverse cosine are $0 \leq \theta \leq \pi$, and because we need to get a negative cosine value, we must be Quadrant II. The angle in Quadrant II that gives us a $\cos(\theta) = -\frac{\sqrt{3}}{2}$ is when $\theta = \frac{5\pi}{6}$.

Now it's your turn. Inverse Trig (Trig)

a. $\tan^{-1}(\cos \pi)$

b. $\arcsin\left(\tan\left(\frac{5\pi}{4}\right)\right)$

c. $\cos^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$

Now it's your turn. Trig (Inverse Trig)

a. $\tan\left(\sin^{-1}\left(\frac{5}{13}\right)\right)$

b. $\cos\left(\arctan\left(-\frac{24}{7}\right)\right)$

c. $\sin(\tan^{-1} x)$ for $x > 0$ (*Challenging problem*)