Math 1131 Review for Midterm 2 (12.1-13.5)

1. Find the derivative of each of the following. Do not simplify.

   a. $f(t)=\log_8(x^3+11)$

   b. $y=x^5\ln x+e^x+x$

   c. $g(x)=e^{x^2-4x+3}+\dfrac{1}{e^{\sqrt{2x-3}}}$

   d. $h(z)=z^3-3^z$

   e. $y=\ln\left(x^{5/4}+x^2-3x\right)$

   f. $y=x^3e^x$

   g. $y=\sqrt{\ln\left(9x^2+3x+2\right)}$

   h. $y=\ln\left(\sqrt{9x^2+3x+2}\right)$

2. Use implicit differentiation to find $\dfrac{dy}{dx}$ from $x^4+e^{xy}+y^2=15$.

3. The equation of the tangent line to the curve $y=(3x-1)^5$ at the point where $x=1$.

4. Find the equation of the tangent line to the graph of $y=e^{3x}+2\ln(x+1)$ at the point (0,1).

5. Use logarithmic differentiation to find the derivative of the following functions.

   a) $\dfrac{d}{dx}\left(\dfrac{\left(x^2+e^x\right)^{10}}{\sqrt[3]{e^{\pi x}}}\right)$

   b) $\dfrac{d}{dx}\left(x^{x^2}\right)$

6. Find Δy and *dy* for the given value of *x* and *dx*.

   $y=\left(x^2+5\right)^2;\ x=1, dx=.5$

7. Approximate $\sqrt{10}$ using differentials.

8. Let $f(x)=x-\ln\left(x^2\right)$.

   a. Find the critical values of $f(x)$.

   b. Use derivatives and/or a sign chart to determine the intervals where $f(x)$ is increasing and where $f(x)$ is decreasing.

   c. Use the info above to find values of *x* for which $f(x)$ has a local maximum and local minimum.

   d. Find the absolute maximum and minimum value of $f(x)$ on the closed interval $[1,e]$

9. Let $f(x) = x^4 - 4x^3$.
   a. Find the *x*- and *y*- intercept(s).
   b. Use the first derivative test to identify any local maxima and/or local minima.
   c. Use derivatives and/or a sign chart to find intervals where $f(x)$ is concave up and where $f(x)$ is concave down.
   d. Find all points of inflection.
   e. Sketch the graph of $y = f(x)$.

10. Let $f(x) = -x^3 + 3x^2 + 1$
   a. Find the critical values of $f(x)$.
   b. Use the Second Derivative Test to find where the local maximum and minimum values of $f(x)$ occur.
   c. Find the absolute maximum and absolute minimum values of $f(x)$ on [-2, 2].

11. Find all *x*-intercepts and *y*-intercepts and give the equations for the vertical and horizontal asymptotes for

   a) $y = \dfrac{3x}{x^2 - 16}$ b) $y = \dfrac{x^2 - 1}{x^2 - 2x - 3}$

12. Sketch a graph of $f(x)$ with the following properties:
   - $f(-7) = f(-4) = f(1) = f(6) = 0$, $f(-2) = 4$, $f(4) = -2$, $f'(-2) = f'(4) = f''(0) = 0$
   - $\lim_{x\to-\infty} f(x) = -2$, $\lim_{x\to\infty} f(x) = 2$, $\lim_{x\to-5^-} f(x) = \infty$, $\lim_{x\to-5^+} f(x) = -\infty$
   - $f'(x) > 0$ on $(-\infty,-5)\cup(-5,-2)\cup(4,\infty)$, $f'(x) < 0$ on $(-2,4)$
   - $f''(x) > 0$ on $(-\infty,-5)\cup(0,6)$, $f''(x) < 0$ on $(-5,0)\cup(6,\infty)$

Answers:

1. a. $f'(t) = \dfrac{3x^2}{\ln 8(x^3 + 11)}$
   b. $y' = 5x^4 \ln x + x^4 + e^x + 1$
   c. $g'(x) = e^{x^2-4x+3}(2x-4) + e^{-\sqrt{2x-3}}\left(-\frac{1}{2}(2x-3)^{-1/2}2\right)$
   d. $h'(z) = 3z^2 - 3^z \ln 3$
   e. $y' = \dfrac{\frac{5}{4}x^{1/4} + 2x - 3}{x^{5/4} + x^2 - 3x}$
   f. $y' = 3x^2e^x + x^3e^x$
   g. $y' = \dfrac{1}{2}\left(\ln\left(9x^2 + 3x + 2\right)\right)^{-1/2}\left(\dfrac{18x+3}{9x^2+3x+2}\right)$

h. $y' = \dfrac{18x+3}{2(9x^2+3x+2)}$

2. $\dfrac{dy}{dx} = \dfrac{-4x^3 - ye^{xy}}{xe^{xy}+2y}$

3. $y = 240x - 208$

4. $y = 5x+1$

5. a) $\dfrac{dy}{dx} = \left(10\dfrac{\left(2x+e^x\right)}{\left(x^2+e^x\right)} - \dfrac{\pi}{3}\right)\cdot\left(\dfrac{\left(x^2+e^x\right)^{10}}{\sqrt[3]{e^{\pi x}}}\right)$

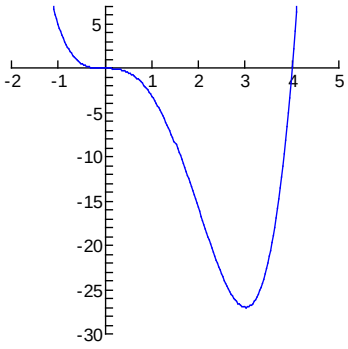
b) $\dfrac{dy}{dx} = x^{x^2}\left(2x\ln x + x\right)$

6. $\Delta y = 16\frac{9}{16},\ dy = 24$

7. $3\frac{1}{6}$

8. a. $x = 2$ b. increasing: $(-\infty, 0)\cup(2, \infty)$; decreasing: (0, 2)
c. no local max; local min at $x = 2$ d. minimum $f(2) = 2 - \ln(4)$, maximum $f(e) = e - 1$

9. a. *x*-int: (0, 0) and (4, 0); *y*-int: (0,0)
b. increasing: (3, ∞); decreasing: ($-\infty$, 3), local min at (3, -27); no local maxima
c. concave up: ($-\infty$, 0) $\cup$ (2, ∞); concave down: (0, 2)
d. (0, 0) and (2, -16)
e. (see graph on the right)



10. a. $x = 0$ and $x = 2$ b. local max at $x = 2$; local min at $x = 0$
c. absolute max is 21 at $x = -2$, absolute min is 1 at $x = 0$

11. a) Intercepts: (0,0); vertical asymptotes $x = 4$ and $x = -4$; horizontal asymptote $y = 0$
b) Intercepts: (1,0), (0,1/3); vertical asymptotes $x = 3$; horizontal asymptote $y = 1$

12.