Math 1131 Review for Midterm 2
(derivatives of logs and exp through applied max and min)

1. Find the derivative of each of the following. Do not simplify.
   a. \( f(t) = \log_8(x^3 + 11) \)
   b. \( y = x^5 \ln x + e^x + x \)
   c. \( g(x) = e^{x^2-4x+3} + \frac{1}{e^{4x^2+3}} \)
   d. \( h(z) = z^3 - 3z \)
   e. \( y = \ln\left(x^{5/4} + x^2 - 3x\right) \)
   f. \( y = x^3 e^x \)
   g. \( y = \sqrt{\ln\left(9x^2 + 3x + 2\right)} \)
   h. \( y = \ln\left(\sqrt{9x^2 + 3x + 2}\right) \)

2. Use implicit differentiation to find \( \frac{dy}{dx} \) from \( x^4 + e^{y^2} + y^2 = 15 \).

3. The equation of the tangent line to the curve \( y = (3x - 1)^3 \) at the point where \( x = 1 \).

4. Find the equation of the tangent line to the graph of \( y = e^{3x} + 2 \ln(x+1) \) at the point (0,1).

5. Use logarithmic differentiation to find the derivative of the following functions.
   a) \( \frac{d}{dx}\left(\frac{(x^2 + e^x)^{10}}{\sqrt[e]{e^{-x}}}\right) \)
   b) \( \frac{d}{dx}(x^2) \)

6. Find \( \Delta y \) and \( dy \) for the given value of \( x \) and \( dx \).
   \( y = (x^2 + 5)^2; \ x = 1, dx = .5 \)

7. Approximate \( \sqrt{10} \) using differentials.

8. Let \( f(x) = x - \ln(x^2) \).
   a. Find the critical values of \( f(x) \).
   b. Use derivatives and/or a sign chart to determine the intervals where \( f(x) \) is increasing and where \( f(x) \) is decreasing.
   c. Use the info above to find values of \( x \) for which \( f(x) \) has a local maximum and local minimum.
   d. Find the absolute maximum and minimum value of \( f(x) \) on the closed interval \([1, e]\)
9. Let \( f(x) = x^4 - 4x^3 \).
   a. Find the x- and y- intercept(s).
   b. Use the first derivative test to identify any local maxima and/or local minima.
   c. Use derivatives and/or a sign chart to find intervals where \( f(x) \) is concave up and where \( f(x) \) is concave down.
   d. Find all points of inflection.
   e. Sketch the graph of \( y = f(x) \).

10. Let \( f(x) = -x^3 + 3x^2 + 1 \)
    a. Find the critical values of \( f(x) \).
    b. Use the Second Derivative Test to find where the local maximum and minimum values of \( f(x) \) occur.
    c. Find the absolute maximum and absolute minimum values of \( f(x) \) on \([-2, 2]\).

11. Find all x-intercepts and y-intercepts and give the equations for the vertical and horizontal asymptotes for
    a) \( y = \frac{3x}{x^2 - 16} \)
    b) \( y = \frac{x^2 - 1}{x^2 - 2x - 3} \)

12. Sketch a graph of \( f(x) \) with the following properties:
    - \( f(-7) = f(-4) = f(1) = f(6) = 0 \), \( f(-2) = 4 \), \( f(4) = -2 \), \( f'(2) = f'(4) = f''(0) = 0 \)
    - \( \lim_{x \to -\infty} f(x) = -2 \), \( \lim_{x \to 3} f(x) = 2 \), \( \lim_{x \to -\infty} f(x) = \infty \), \( \lim_{x \to 5} f(x) = -\infty \)
    - \( f'(x) > 0 \) on \((-\infty, -5) \cup (-3, -2) \cup (4, \infty)\), \( f'(x) < 0 \) on \((-2, 4)\)
    - \( f''(x) > 0 \) on \((-\infty, -5) \cup (0, 6)\), \( f''(x) < 0 \) on \((-5, 0) \cup (6, \infty)\)

13. A company manufactures and sells \( x \) mp3 players per week. If the weekly cost and demand equations are given by: \( C(x) = 8000 + 5x \) and \( p = 14 - \frac{x}{4000}, \quad 0 \leq x \leq 25,000 \)
    Find the production level that maximizes profit.

14. A company wishes to manufacture a box with a volume of 36 cubic feet that is open on top and is twice as long as it is wide. Find the width of the box that can be produced using the minimum amount of material.

15. A real estate firm owns 100 apartments. At $400 per month, each apartment can be rented. However, for each $10-per-month increase, there will be two vacancies with no possibility of filling them. What rent per apartment will maximize the monthly revenue?
Answers:

1. \( f'(t) = \frac{3x^2}{\ln(8x^3 + 11)} \)
   b. \( y' = 5x^4 \ln x + x^4 + e^x + 1 \)
   c. \( g'(x) = e^{x^2-4x+3} (2x-4) + e^{-\sqrt{2x-3}} (-\frac{1}{2} (2x-3)^{3/2}) \)
   d. \( h'(z) = 3z^2 - 3z \ln 3 \)
   e. \( y' = \frac{5}{x^{1/4} + 2x - 3} \)
   f. \( y' = 3x^2 e^x + x^3 e^x \)
   g. \( y' = \frac{1}{2} (\ln (9x^2 + 3x + 2))^{-1/2} \left( \frac{18x + 3}{9x^2 + 3x + 2} \right) \)
   h. \( y' = \frac{18x + 3}{2(9x^2 + 3x + 2)} \)

2. \( \frac{dy}{dx} = \frac{-4x^3 - ye^{xy}}{xe^{xy} + 2y} \)
3. \( y = 240x - 208 \)
4. \( y = 5x + 1 \)

5. a) \( \frac{dy}{dx} = \left( 10 \left( \frac{2x + e^x}{x^2 + e^x} \right) - \frac{\pi}{3} \right) \left( \frac{x^2 + e^x}{\sqrt[3]{e^{2x}}} \right) \)
   b) \( \frac{dy}{dx} = x^2 \left( 2x \ln x + x \right) \)

6. \( \Delta y = 16 \frac{\pi}{16}, \ dy = 24 \)
7. \( 3 \frac{1}{2} \)
8. a. \( x = 2 \)  b. increasing: \((-\infty, 0) \cup (2, \infty)\); decreasing: \((0, 2)\)
   c. no local max; local min at \( x = 2 \)  d. minimum \( f(2) = 2 - \ln(4) \), maximum \( f(e) = e - 1 \)

9. a. \( x\)-int: \((0, 0)\) and \((4, 0)\); \( y\)-int: \((0,0)\)
   b. increasing: \((3, \infty)\); decreasing: \((-\infty, 3)\), local min at \( (3, -27) \); no local maxima
   c. concave up: \((-\infty, 0) \cup (2, \infty)\); concave down: \((0, 2)\)
   d. \((0, 0)\) and \((2, -16)\)
   e. (see graph on the right)

10. a. \( x = 0 \) and \( x = 2 \)  b. local max at \( x = 2 \); local min at \( x = 0 \)
   c. absolute max is 21 at \( x = -2 \), absolute min is 1 at \( x = 0 \)

11. a) Intercepts: \((0,0)\); vertical asymptotes \( x = 4 \) and \( x = -4 \); horizontal asymptote \( y = 0 \)
   b) Intercepts: \((1,0), (0,1/3)\); vertical asymptotes \( x = 3 \); horizontal asymptote \( y = 1 \)
12. 

13. 18,000 mp3 players per week

14. 3 feet

15. $450