Math 1131 Review for the Final Exam

1. Find the following limits:
   a. \( \lim_{x \to 3} \frac{x^2 - 5x + 6}{x^3 - 3x^2} \)
   b. \( \lim_{x \to 2} \frac{x^2 - 3x}{x^2 - 4} \)
   c. \( \lim_{x \to \infty} \frac{7 - 16x - 8x^2}{5x^2 + 2x - 3} \)
   d. \( \lim_{x \to 0} \frac{\sqrt{x + 4} - 2}{x} \)
   e. \( \lim_{h \to 0} \frac{\frac{5}{x+h} - \frac{5}{x}}{h} \)

2. Let \( f(x) = \begin{cases} 
3x + 2 & \text{if } x \leq -2 \\
7x + 7 & \text{if } -2 < x < 1 \\
-\frac{32x}{x-2(x+3)} & \text{if } x \geq 1
\end{cases} \)
   a. Find \( \lim_{x \to -2} f(x) \).
   b. Find \( \lim_{x \to -2} f(x) \).
   c. Find all \( x \) for which \( f(x) \) is not continuous.

3. Solve the inequality: \( \frac{x^4(x - 2)}{x + 5} \leq 0 \).

4. Use the definition of derivative to find \( f'(x) \) if \( f(x) = x^2 - 4x \).

5. Find \( \frac{dy}{dx} \) for each of the following:
   a. \( y = e^{3x^4 + 2x + 5} \)
   b. \( y = (2x - 6)^2 \sqrt{x^3 + 5x - 1} \)
   c. \( y = \frac{(3x - 7)^2}{(4x + 9)^3} \)
   d. \( y = \left( \ln(5x^2 - 3x + 1) \right)^3 - (6x + 1)^{3/4} \)

6. Use implicit differentiation to find \( \frac{dy}{dx} \) in terms of \( x \) and \( y \), where \( x^4 + e^{3y} + y^3 = -42 \).

7. Find the equation of the tangent line to the graph of \( y = 4 \ln(2x - 5) - 2x \) at the point \((3, -6) \).

8. Use the Second Derivative Test to find the x-values of the relative extrema of the function \( f(x) = x^3 - 3x^2 - 4 \).

9. Let \( f(x) = x^3 + 6x^2 + 9x \).
   a. Find the x- and y-intercepts of \( f(x) \).
   b. Use derivatives and/or a sign chart to find the interval(s) where \( f(x) \) is increasing and where \( f(x) \) is decreasing.
   c. Use the information in (b) to find all point(s) of relative max and relative min.
   d. Use derivatives and/or a sign chart to find the interval(s) where \( f(x) \) is concave up and where \( f(x) \) is concave down.
   e. Use the information in (d) to find all point(s) of inflection.
   f. Sketch the graph of \( y = f'(x) \), clearly showing the information found in (a) – (e).
10. Find the vertical and horizontal asymptotes for \( f(x) = \frac{3x + 5}{x^2 - 2x - 8} \).

11. A deli sells 640 sandwiches per day at a price of $8 each. A market survey shows that for every $0.10 reduction in price, 40 more sandwiches will be sold. How much should the deli charge for a sandwich in order to maximize revenue?

12. Let the cost of producing \( x \) Whosy-Whatsits be modeled by the equation \( C(x) = 15x^3 + 100x^2 + 6000x + 5000 \)

\[ R(x) = 5x^3 + 400x^2 + 30000x \]

a. Compute the marginal cost, marginal revenue, and marginal profit functions.
b. Find the revenue associated with the maximum profit

13. Evaluate the following integrals:

\[
\begin{align*}
\text{a. } & \int_4^7 \left( \frac{1}{\sqrt{6x}} + \sqrt{3} \right) dx \\
\text{b. } & \int xe^{3x^2 + 11} dx \\
\text{c. } & \int \frac{\ln x}{x} dx \\
\text{d. } & \int_0^2 \frac{e^{3x}}{2e^{3x} + 5} dx \\
\text{e. } & \int 3x^3 \left( \sqrt{x^4 + 12} \right) dx \\
\end{align*}
\]

14. Find an approximate area \( S_4 \) of the region bounded by the given curves in the first quadrant. (Use the right-hand endpoint of each subinterval.) \( f(x) = x^3 + x; \quad y = 0; \quad x = 8 \)

15. Find the area of the region between \( y = x^3 - 9x \) and \( y = x \).

16. The demand for a product is given by \( p = 500 - q^2 \) and its supply is given by \( p = 30q + 100 \). Find the consumers' surplus under market equilibrium.
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Answers

1. a. 1/9  b. $-\infty$  c. -8/5  d. $\frac{1}{4}$  e. $\frac{-5}{x^2}$

2. a. -4  b. 5  c. x = -2, 2

3. (-5, 2)

4. 2x - 4

5. a. $e^{3x^2 + 2x + 5} \left( 12x^3 + 2 \right)$
   b. $5 \left( 2x - 6 \right)^4 \left( 2 \right) (x^3 + 5x - 1)^{1/2} + \left( 2x - 6 \right)^5 \left( 1/2 \right) (x^3 + 5x - 1)^{-1/2} (3x^2 + 5)$
   c. $\frac{2(3x - 7)(3)(4x + 9)^3 - (3x - 7)^2 (3)(4x + 9)^2 (4)}{(4x + 9)^6}$
   d. $3 \left( \ln \left( 5x^2 - 3x + 1 \right) \right)^2 \frac{1}{5x^2 - 3x + 1} (10x - 3) - \frac{3}{4} \left( 6x + 1 \right)^{-1/4} (6)$

6. $\frac{-4x^3 - ye^{xy}}{xe^{xy} + 3y^2}$

7. $y = 6 \left( x - 3 \right) - 6$

8. rel max at x = 0, rel min at x = 2

9. a. (0, 0) and (-3, 0)  b. inc: (-∞, -3) and (-1, ∞); dec: (-3, -1)
   c. rel max at x = -3, rel min at x = -1  d. CU: (-2, ∞), CD: (-∞, -2)
   e. inf pt at x = -2

10. Vertical asymptotes x = 4 and x = -2, Horizontal asymptote y = 0

11. $\$4,800$

12. a) $C'(x) = 45x^2 + 200x + 6000$
   b) $R'(x) = 15x^2 + 800x + 30000$
   c) $P'(x) = -30x^2 + 600x + 24000$

13. a) $\approx 6.1368$  b) $\frac{1}{8} e^{4x^2 + 11} + C$
   c) $\left( \frac{\ln x}{6} \right)^6 + C$
   d) $\frac{\ln \left( 2e^6 + 5 \right)}{6} - \frac{\ln \left( 7 \right)}{6}$
   e) $\frac{1}{2} \left( x^4 + 12 \right)^{3/2} + C$

14. 1640

15. 50

16. $\frac{2000}{3} \approx 666.67$