Math 1131 Review for the Final Exam

1. Find the following limits:

   a. $\lim_{x\to 3}\dfrac{x^2-5x+6}{x^3-3x^2}$ b. $\lim_{x\to 2^+}\dfrac{x^2-3x}{x^2-4}$ c. $\lim_{x\to\infty}\dfrac{7-16x-8x^2}{5x^2+2x-3}$

   d. $\lim_{x\to 0}\dfrac{\sqrt{x+4}-2}{x}$ e. $\lim_{h\to 0}\dfrac{\frac{5}{x+h}-\frac{5}{x}}{h}$

2. Let $f(x)=\begin{cases}3x+2 & \text{if } x\le -2\\ x+7 & \text{if } -2<x<1\\ \frac{-32x}{(x-2)(x+3)} & \text{if } x\ge 1\end{cases}$.

   a. Find $\lim_{x\to -2^-} f(x)$. b. Find $\lim_{x\to -2^+} f(x)$ c. Find all *x* for which $f(x)$ is not continuous.

3. Solve the inequality: $\dfrac{x^4(x-2)}{x+5}\le 0$.

4. Use the definition of derivative to find $f'(x)$ if $f(x)=x^2-4x$.

5. Find $\frac{dy}{dx}$ for each of the following:

   a. $y=e^{3x^4+2x+5}$ b. $y=(2x-6)^5\sqrt{x^3+5x-1}$

   c. $y=\dfrac{(3x-7)^2}{(4x+9)^3}$ d. $y=\left(\ln\left(5x^2-3x+1\right)\right)^3-(6x+1)^{3/4}$

6. Use implicit differentiation to find $\frac{dy}{dx}$ in terms of *x* and *y*, where $x^4+e^{xy}+y^3=-42$.

7. Find the equation of the tangent line to the graph of $y=4\ln(2x-5)-2x$ at the point (3, -6).

8. Use the Second Derivative Test to find the x-values of the relative extrema of the function $f(x)=x^3-3x^2-4$.

9. Let $f(x)=x^3+6x^2+9x$.

   b. Find the *x*- and *y*-intercepts of *f(x).*

   c. Use derivatives and/or a sign chart to find the interval(s) where *f(x)* is increasing and where *f(x)* is decreasing.

   d. Use the information in (b) to find all point(s) of relative max and relative min.

   e. Use derivatives and/or a sign chart to find the interval(s) where *f(x)* is concave up and where *f(x)* is concave down.

   f. Use the information in (d) to find all point(s) of inflection.

   g. Sketch the graph of $y=f(x)$, clearly showing the information found in (a) – (e).

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10. Find the vertical and horizontal asymptotes for $f(x)=\dfrac{3x+5}{x^2-2x-8}$.

11. A deli sells 640 sandwiches per day at a price of \$8 each. A market survey shows that for every \$0.10 reduction in in price, 40 more sandwiches will be sold. How much should the deli charge for a sandwich in order to maximize revenue?

12. Let the cost of producing x Whosy-Whatsits be modeled by the equation C (*x*) and the revenue from selling x Whosy-Whatsits be modeled by *R* (*x*).

$$C(x)=15x^3+100x^2+6000x+5000$$

$$R(x)=5x^3+400x^2+30000x$$

    a. Compute the marginal cost, marginal revenue, and marginal profit functions.
    b. Find the revenue associated with the maximum profit

13. Evaluate the following integrals:

a. $\displaystyle\int_4^7\left(\frac{1}{\sqrt[3]{6x}}+\sqrt{3}\right)dx$

b. $\displaystyle\int xe^{4x^2+11}dx$

c. $\displaystyle\int\frac{(\ln x)^5}{x}dx$

d. $\displaystyle\int_0^2\frac{e^{3x}}{2e^{3x}+5}dx$

e. $\displaystyle\int 3x^3\left(\sqrt{x^4+12}\right)dx$

14. Find an approximate area S_4 of the region bounded by the given curves in the first quadrant. (Use the right-hand endpoint of each subinterval.) $f(x)=x^3+x;\quad y=0;\quad x=8$

15. Find the area of the region between $y=x^3-9x$ and $y=x$.

16. The demand for a product is given by $p=500-q^2$ and its supply is given by $p=30q+100$. Find the consumers' surplus under market equilibrium.

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Answers

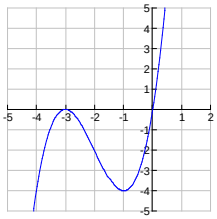
1. a. 1/9 b. $-\infty$ c. -8/5 d. $\frac{1}{4}$ e. $\frac{-5}{x^2}$
2. a. -4 b. 5 c. x = -2, 2
3. (-5, 2]
4. $2x-4$
5. a. $e^{3x^4+2x+5}\left(12x^3+2\right)$

   b. $5\left(2x-6\right)^4\left(2\right)\left(x^3+5x-1\right)^{1/2}+\left(2x-6\right)^5\left(1/2\right)\left(x^3+5x-1\right)^{-1/2}\left(3x^2+5\right)$

   c. $\dfrac{2\left(3x-7\right)\left(3\right)\left(4x+9\right)^3-\left(3x-7\right)^2\left(3\right)\left(4x+9\right)^2\left(4\right)}{\left(4x+9\right)^6}$

   d. $3\left(\ln\left(5x^2-3x+1\right)\right)^2\dfrac{1}{5x^2-3x+1}\left(10x-3\right)-\dfrac{3}{4}\left(6x+1\right)^{-1/4}\left(6\right)$

6. $\dfrac{-4x^3-ye^{xy}}{xe^{xy}+3y^2}$
7. $y=6\left(x-3\right)-6$
8. rel max at x = 0, rel min at x = 2
9. a. (0. 0) and (-3, 0) b. inc: ($-\infty$, -3) and (-1, ∞); dec: (-3, -1)
   c. rel max at x = -3, rel min at x = -1 d. CU: (-2, ∞), CD: ($-\infty$, -2)
   e. inf pt at x = -2 f.



10. Vertical asymptotes x = 4 and x = -2, Horizontal asymptote y = 0
11. $4.80
12. a) b) $2,160,000

$C'(x)=45x^2+200x+6000$

$R'(x)=15x^2+800x+30000$

$P'(x)=-30x^2+600x+24000$

13. a) ≈ 6.1368 b) $\frac{1}{8}e^{4x^2+11}+C$ c) $\dfrac{\left(\ln x\right)^6}{6}+C$

    d) $\dfrac{\ln\left(2e^6+5\right)}{6}-\dfrac{\ln\left(7\right)}{6}$ e) $\frac{1}{2}\left(x^4+12\right)^{3/2}+C$

14. 1640
15. 50
16. $\dfrac{2000}{3}\approx 666.67$