a) \[
\lim_{x \to 3} \frac{x^2 - 5x + 6}{x^3 - 3x^2} = 0
\]
\[
= \lim_{x \to 3} \frac{(x-3)(x-2)}{x^2(x-3)}
\]
Now plug in \(x = 3\):
\[
\frac{3-2}{3^2} = \frac{1}{9}
\]

b) \[
\lim_{x \to 2^+} \frac{x^2 - 3x}{x^2 - 4} = \frac{\infty}{0} \quad \text{The answer is } +\infty \text{ or } -\infty. \text{ Make a sign chart!}
\]
\[
\begin{array}{c|c|c|c}
\text{Sign Chart} & \text{Sign} & \text{Reason} \\
------------------- & \hline
x^2 - 4 & - & \text{Factor: } (x-2)(x+2) \\
x^2 - 3x & + & \\
\hline
x & - & \text{Root: } 2 \\
0 & + & \text{Root: } \frac{3}{2} \\
2 & - & \text{Root: } 4 \\
\hline
\end{array}
\]
⇒ The limit is \(-\infty\).
Use this since \(x \to 2^+\).

c) \[
\lim_{x \to \infty} \frac{7-16x-8x^2}{5x^3 - 2x - 3} = \lim_{x \to \infty} \frac{-8x^2}{5x^3} = \frac{-8}{5}
\]

d) \[
\lim_{x \to 0} \frac{\sqrt{x+4} - 2}{x}
\]
\[
= \lim_{x \to 0} \frac{\sqrt{x+4} - 2}{x} \cdot \frac{\sqrt{x+4} + 2}{\sqrt{x+4} + 2}
\]
\[
= \lim_{x \to 0} \frac{x+4}{x(\sqrt{x+4} + 2)}
\]
Plug \(x = 0\):
\[
= \frac{1}{4}
\]

e) \[
\lim_{h \to 0} \frac{\frac{5}{h} - \frac{6}{x}}{h}
\]
\[
= \lim_{h \to 0} \frac{1}{h} \left( \frac{5}{x} - \frac{6}{x} \right)
\]
\[
= \lim_{h \to 0} \frac{1}{h} \left( \frac{5x - 6(x+h)}{x(x+h)} \right)
\]
\[
= \lim_{h \to 0} \frac{1}{h} \left[ \frac{5x - 6(x+h)}{x(x+h)} \right]
\]
Plug in \(h > 0\):
\[
\lim_{h \to 0} \frac{-5x - 6(x+h)}{x(x+h)} = -\frac{2}{x^2}
\]
\[ f(x) = \begin{cases} 
3x+2, & x \leq -2 \\
x+7, & -2 < x \leq 1 \\
-\frac{3x}{(x+2)(x+3)}, & x > 1 
\end{cases} \]

a) \[ \lim_{{x \to -2^-}} f(x) = \lim_{{x \to -2^-}} (3x+2) = 3(-2)+2 = -4 \]

b) \[ \lim_{{x \to 1^+}} f(x) = \lim_{{x \to 1^+}} (x+7) = -2+7 = 5 \]

c) Find all \( x \) where \( f(x) \) is discontinuous.

Two things to check:
1. Find where each piece has a discontinuity.
   - The top and middle pieces are polynomials \( \Rightarrow \) they have no discontinuities.
   - The bottom piece is a rational function. It has discontinuities if \( \text{denom} = 0 \).
     \[ (x-2)(x+3) = 0 \]
     \( \Rightarrow \) \( x = 2 \) or \( x = -3 \)
     
     We only use the bottom piece \( \forall \, x < 1 \), so ignore \( x = -3 \).
     
     The only discontinuity in the separate pieces is \( x = 2 \)

2. Check the endpoints \( x = -2 \) and \( x = 1 \) separately.

   We need to check 3 things:
   1) \( f(a) \) is defined
   2) \( \lim_{{x \to a^-}} f(x) = \lim_{{x \to a^+}} f(x) \) \( \quad \text{If any of these fail, the function is discontinuous at } x = a \).
   3) \( f(a) = \lim_{{x \to a^-}} f(x) = \lim_{{x \to a^+}} f(x) \) \( \quad \text{If any of these fail, the function is discontinuous at } x = a \).

   - For \( x = -2 \): From parts a), b), we see \( \lim_{{x \to -2^-}} f(x) \neq \lim_{{x \to -2^+}} f(x) \)
     
     So, \( f(x) \) is discontinuous at \( x = -2 \).

   - For \( x = 1 \):
     1) Is \( f(1) \) defined? Yes since \( f(1) = \frac{-32(1)}{(4-2)(4+3)} = -8 \)
     2) Is \( \lim_{{x \to 1^-}} f(x) = \lim_{{x \to 1^+}} f(x) \)? Yes!
        \[ \lim_{{x \to 1^-}} f(x) = \lim_{{x \to 1^-}} (x+7) = 8 \quad \lim_{{x \to 1^+}} f(x) = \lim_{{x \to 1^+}} \left( -\frac{3x}{(x+2)(x+3)} \right) = 8 \]
     3) Is \( f(1) = \lim_{{x \to 1^-}} f(x) = \lim_{{x \to 1^+}} f(x) \)? Yes (both = 8).

     So, \( f(x) \) is continuous at \( x = 1 \)!

Thus, the only discontinuities of \( f(x) \) are \( x = 2, x = -1 \).
\[
\frac{x^4(x-2)}{x+5} \leq 0.
\]

Make a sign chart:

\[
\begin{array}{c|c|c|c|c}
& + & - & + & - \\
\hline
x & + & - & + & - \\
\hline
& (1) 0 & - & 5 & (1) 0 \\
\hline
& (1) 0 & - & 0 & (1) 0 \\
\hline
& (1) 0 & - & 2 & (1) 0 \\
\hline
& (1) 0 & - & 3 & (1) 0 \\
\end{array}
\]

We want the function to be \( \leq 0 \), so the regions:

\[
[-5, 0] \cup [0, 2] \quad \text{or} \quad (-\infty, 2]
\]
Use the defn of a deriv. to find \( f'(x) \) if \( f(x) = x^2 - 4x \).

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

To find \( f(x+h) \):

1) First rewrite \( f(x) \) by placing \( () \) around each \( x \)

\[
f(x) = (x^2 - 4x)
\]

2) Replace each \( x \) with \( x+h \)

\[
f(x+h) = ((x+h)^2 - 4(x+h))
\]

\[
x^2 + 2xh + h^2 - 4(x + h)
\]

\[
x^2 + 2xh + h^2 - 4x - 4h
\]

Plug in: \( f'(x) = \lim_{h \to 0} \frac{[x^2 + 2xh + h^2 - 4x - 4h] - [x^2 - 4x]}{h}
\]

\[
= \lim_{h \to 0} \frac{2xh + h^2 - 4h}{h}
\]

\[
= \lim_{h \to 0} \frac{2x + h - 4}{h}
\]

Plug in \( h=0 \):

\[
2x - 4
\]
a) \[ y = e^{3x^4 + 2x + 5} \]
\[ y' = (3x^4 + 2x + 5) e^{3x^4 + 2x + 5} \]
\[ y' = e^{3x^4 + 2x + 5} \left( \frac{3x^4 + 2x + 5}{12x^3 + 2} \right) \]

b) \[ y = (2x-6)^5 (x^3 + 5x - 1)^{1/2} \]

Using the product rule:
\[ y' = \left[ (2x-6)^5 \right]' (x^3 + 5x - 1)^{1/2} + (2x-6)^5 \left[ (x^3 + 5x - 1)^{1/2} \right]' \]
\[ y' = [5(2x-6)^4 (2x-6)^5] (x^3 + 5x - 1)^{1/2} + (2x-6)^5 \left( \frac{1}{2} (x^3 + 5x - 1)^{-1/2} (3x^2 + 5) \right) \]

\[ y' = 5(2x-6)^4 (2x-6)^5 (x^3 + 5x - 1)^{1/2} + (2x-6)^5 \cdot \frac{1}{2} (x^3 + 5x - 1)^{-1/2} (3x^2 + 5) \]

Cc) \[ y = \frac{(3x-7)^2}{(4x+9)^3} \]

Using the quotient rule:
\[ y' = \frac{[(3x-7)^2]' (4x+9)^3 - (3x-7)^2 [4x+9]^3]'}{[(4x+9)^3]^2} \]
\[ y' = \frac{2(3x-7)(3)(4x+9)^3 - (3x-7)^2 \cdot 3(4x+9)^2 (4)}{(4x+9)^6} \]

\[ y' = \frac{2(3x-7)(3)(4x+9)^3 - (3x-7)^2 \cdot 3(4x+9)^2 (4)}{(4x+9)^6} \]

\[ y' = \frac{2(3x-7)(3)(4x+9)^3 - (3x-7)^2 \cdot 3(4x+9)^2 (4)}{(4x+9)^6} \]

\[ y' = \frac{2(3x-7)(3)(4x+9)^3 - (3x-7)^2 \cdot 3(4x+9)^2 (4)}{(4x+9)^6} \]

\[ y' = \frac{2(3x-7)(3)(4x+9)^3 - (3x-7)^2 \cdot 3(4x+9)^2 (4)}{(4x+9)^6} \]

\[ y' = \frac{2(3x-7)(3)(4x+9)^3 - (3x-7)^2 \cdot 3(4x+9)^2 (4)}{(4x+9)^6} \]

\[ y' = \frac{2(3x-7)(3)(4x+9)^3 - (3x-7)^2 \cdot 3(4x+9)^2 (4)}{(4x+9)^6} \]
\[ x^4 + e^{xy} + y^3 = -4y. \]

**Step 1:** Take \( \frac{d}{dx} \) of both sides.

\[
\frac{d}{dx}(x^4 + e^{xy} + y^3) = \frac{d}{dx}(-4y).
\]

\[
4x^3 + e^{xy}(x'y + yx') + \frac{d}{dx}(y^3) = 0
\]

\[
4x^3 + e^{xy}[\frac{d}{dx}(y) + x \cdot y'] + 3y^2 y' = 0
\]

\[
4x^3 + e^{xy}[y + x \cdot y'] + 3y^2 y' = 0.
\]

**Step 2:** Distribute (if necessary):

\[
4x^3 + ye^{xy} + xue^{xy} y' + 3y^2 y' = 0
\]

**Step 3:** Get all terms w/ \( y' \) on LHS, all others on RHS:

\[
xue^{xy} y' + 3y^2 y' = -4x^3 - ye^{xy}
\]

**Step 4:** Factor out \( y' \) from LHS:

\[
(xe^{xy} + 3y^2) y' = -4x^3 - ye^{xy}
\]

**Step 5:** Divide to get \( y' \)

\[
y' = \frac{-4x^3 - ye^{xy}}{xe^{xy} + 3y^2}
\]
Find the tangent line to \( y = 4 \ln (2x-5) - 2x \) at \((3, -6)\).

1) Find \( y' \)
\[ y' = 4 \cdot \frac{1}{2x-5} - 2 \]

2) Plug in \( x = 3 \), \( y = -6 \) to \( y' \)
\[ y' = 4 \cdot \frac{1}{2(3)-5} - 2 \]
\[ y' = 4 \cdot \frac{1}{1} - 2 \]
\[ y' = 2 - 2 \]
\[ y' = 0 \]
Thus is the slope of the tan line!

3) Plug into slope form of the line:
\[ y - y_0 = m(x-x_0) \]
\[ y - (-6) = 0(x-3) \]
\[ y + 6 = 0 \]
\[ y = -6 \]
\[ y = 6x - 24 \]
\[ f(x) = x^3 - 3x^2 + 4. \] Find extrema using Second Deriv Test.

**Steps:**

1) Find the critical pts:

\[ f'(x) = 3x^2 - 6x \]

\[ 0 = 3x(x-2) \]

\[ \text{CP: } x = 0, x = 2 \]

2) Find 2nd deriv:

\[ f''(x) = 6x - 6 \]

3) Use the 2nd Deriv Test.

**Reminder:** The 2nd Deriv Test tells us:

If \( x = a \) is a critical pt and:

1. \( f''(a) > 0 \), rel min at \( x = a \)
2. \( f''(a) < 0 \), rel max at \( x = a \)
3. \( f''(a) = 0 \), the test fails.

Check the CP:

- For \( x = 0 \): \( f''(0) = 6(0) - 6 < 0 \) => Rel max at \( x = 0 \)
- For \( x = 2 \): \( f''(2) = 6(2) - 6 > 0 \) => Rel min at \( x = 2 \)
\[ f(x) = x^3 + 6x^2 + 9x \]

b) Find the \( x \) and \( y \) intercepts.

For \( x \) intercepts: set \( y = 0 \)
\[
0 = x^3 + 6x^2 + 9x \\
0 = x(x^2 + 6x + 9) \\
0 = x(x + 3)^2
\]
\( \Rightarrow \) \( x = 0 \), \( x = -3 \)

\( x \) intercepts are \( (0,0) \), \( (-3,0) \)

For \( y \) intercept: set \( x = 0 \) \( \Rightarrow \) \( y \) int. = \( (0,0) \)

c) Find where \( f \) is inc./dec.

\[
f'(x) = 3x^2 + 12x + 9
\]
\[
f'(x) = 3(x^2 + 4x + 3) = 3(x + 3)(x + 1)
\]

Sign chart \( f' \):
\[
\begin{array}{c|ccc}
& (-) & -3 & (+) \\
\hline
f' & (-) & -3 & (+)
\end{array}
\]
\( \Rightarrow \)
\( f \) is inc. on \( (-\infty, -3) \) and \( (-1, \infty) \)
\( f \) is dec. on \( (-3, -1) \)

d) Find rel max and min.

Rel max at \( x = -3 \) \( \Rightarrow \) To find \( y \), plug \( x = 3 \) into the original \( f(x) \):
\[
y = (3)^3 + 6(3)^2 + 9(3) = 0.
\]
Rel max is \( y = 0 \) when \( x = -3 \)

Rel min at \( x = -1 \) \( \Rightarrow \) To find \( y \):
\[
y = (-1)^3 + 6(-1)^2 + 9(-1) = -4.
\]
Rel min is \( y = -4 \) when \( x = -1 \)

e) Find where \( f \) is conc. up/down.

\[
f''(x) = 6x + 12
\]

Sign chart for \( f'' \):
\[
\begin{array}{c|c}
& 6x + 12 \\
\hline
\text{Sign} & (-) \\
\hline
f'' & -2
\end{array}
\]
\( \Rightarrow \)
\( f \) is conc. up on \( (-2, \infty) \)
\( f \) is conc. down on \( (-\infty, -2) \)

f) Find the inflection pts.

Inf pt at \( x = -2 \) \( \Rightarrow \) To find \( y \), plug \( x = -2 \) into the original function:
\[
y = (-2)^3 + 6(-2)^2 + 9(-2) = -8 + 24 - 18 = -2.
\]
The inf. pt. is \( (-2, -2) \)
Find vertical and horizontal asymptotes for

\[ y = \frac{3x+5}{x^2-2x-8} \]

For VA: Factor the expression and cancel like term.

\[ y = \frac{3x+5}{(x-4)(x+2)} \]

The VA are the places the simplified expr is 0 in the denon:

VA: \[ x = 4, \ x = -2 \]

For HA: Take \( x \to \infty \) and \( x \to -\infty \).

\[ \lim_{x \to \infty} \frac{3x+5}{x^2-2x-8} = \lim_{x \to \infty} \frac{3x}{x^2} = \lim_{x \to \infty} \frac{3}{x} = 0 \]

Nothing changes if \( x \to -\infty \), so: HA are \( y = 0 \) as \( x \to \pm \infty \)
1. In order to maximize revenue, I need a revenue function. 
\[ \text{Known: revenue} = \text{price} \times \text{(# of units sold)} \]

Let \( x = \# \text{ of times we reduce the price} \).

\[
\begin{align*}
\text{Price} &= 8 - 0.1x \\
\text{# of sandwiches} &= 640 + 40x \\
\Rightarrow \text{Revenue} &= r = (8 - 0.1x)(640 + 40x) \\
r &= 5120 + 320x - 64x - 4x^2 \\
r &= -4x^2 + 256x + 5120.
\end{align*}
\]

2. Find CP: 
\[ r' = -8x + 256. \]
\[ 0 = -8x + 256 \]
\[ 8x = 256 \]
\[ x = 32 \]

Check this is a max: 
\[ \frown \bigg|_{0}^{32} \bigg| \]
There is an inflection at \( x = 32 \). 
This is also an abs max.

3. Use the CP correctly to ans. the question.
We need to know the price:

\[
\begin{align*}
\text{Price} &= 8 - 0.1x \\
&= 8 - 0.1(32) \\
&= 8 - 3.2 \\
&= \boxed{4.80}
\end{align*}
\]
\[
C(x) = 15x^3 + 100x^2 + 6000x + 5000 \\
R(x) = 5x^3 + 400x^2 + 30000x
\]

a) Find marginal cost, marginal revenue, and marginal profit.

\[
MC = \frac{dC}{dx} = 45x^2 + 200x + 6000
\]

\[
MR = \frac{dR}{dx} = 15x^2 + 800x + 30000
\]

\[
MP = MR - MC = (15x^2 + 800x + 30000) - (45x^2 + 200x + 6000)
\]

\[
= 15x^2 + 800x + 30000 - 45x^2 - 200x - 6000
\]

\[
= -30x^2 + 600x + 24000
\]

b) Find revenue associated to maximum profit.

Find cp of profit by setting \( MP = 0 \).

\[
P' = 0 = -30x^2 + 600x + 24000
\]

\[
0 = x^2 - 20x - 80
\]

\[
0 = (x - 40)(x + 20)
\]

\[
x = 40, -20
\]

Sign chart for \( P' = -30x^2 + 600x + 24000 \)

\[
\begin{array}{c|c|c|c}
\text{Interval} & \text{Test Value} & \text{Sign of } P' & \text{Behavior of } P \\
\hline
0 & 40 & + & \text{Relative Max @ } x=40! \text{ This is also an absolute max.}
\end{array}
\]

To find the revenue associated w/ max profit, we'll plug \( x=40 \) into the revenue function:

\[
R(40) = 5(40)^3 + 400(40)^2 + 30000(40)
\]

\[
= 12,160,000
\]
a) \[ \int_{\frac{1}{4}}^{7} \left( \frac{1}{3} \sqrt[3]{x^2} + \sqrt[3]{x} \right) \, dx = \int_{\frac{1}{4}}^{7} \left( \frac{1}{3} \sqrt[3]{\frac{1}{x^2}} + \sqrt[3]{x} \right) \, dx \]
\[= \int_{\frac{1}{4}}^{7} \left( \frac{1}{3} \cdot \frac{1}{\sqrt[3]{x^2}} + \sqrt[3]{x} \right) \, dx \]
\[= \frac{1}{3} \left[ \frac{1}{\sqrt[3]{x^2}} \cdot \frac{x}{3} + \frac{3}{2} x \right]_{\frac{1}{4}}^{7} \]
\[= \left[ \frac{3}{4} \cdot \frac{1}{\sqrt[3]{x^2}} + \frac{3}{2} x \right]_{\frac{1}{4}}^{7} \]
\[= \left[ \frac{3}{4} \cdot \frac{1}{7^{\frac{2}{3}}} + \frac{3}{2} \cdot 7 \right] - \left[ \frac{3}{4} \cdot \frac{1}{\left(\frac{1}{4}\right)^{\frac{2}{3}}} + \frac{3}{2} \cdot \left(\frac{1}{4}\right) \right] \]
\[\approx 6.1368 \]

b) \[ \int x e^{4x^2 + 11} \, dx \]
\[u = 4x^2 + 11 \]
\[du = 8x \, dx \]
\[\frac{du}{8x} = dx \]
\[\int x e^{4x^2 + 11} \, dx = \int \underbrace{\frac{1}{8} e^u \, du} = \frac{1}{8} \int e^u \, du \]
\[= \frac{1}{8} e^{4x^2 + 11} + C \]

c) \[ \int \frac{(\ln x)^5}{x} \, dx \]
\[u = \ln x \]
\[du = \frac{1}{x} \, dx \]
\[\int \frac{u^5}{x} \, dx = \int u^5 \underbrace{x \, du} = \int u^5 \, du = \frac{1}{6} u^6 + C \]
\[= \frac{1}{6} (\ln x)^6 + C \]

d) \[ \int_{0}^{2} \frac{e^{3x}}{2e^{3x} + 5} \, dx \]
\[u = 2e^{3x} + 5 \]
\[du = 6e^{3x} \, dx \]
\[\frac{1}{6e^{3x}} \, du = dx \]
\[\int_{0}^{2} \frac{e^{3x}}{2e^{3x} + 5} \, dx = \int_{0}^{2} \frac{1}{6} \, du \]
\[= \frac{1}{6} \ln \left( 2e^{3x} + 5 \right) \big|_{x=0}^{x=2} \]
\[= \frac{1}{6} \ln \left( 2e^{6} + 5 \right) - \frac{1}{6} \ln 7 \]

e) \[ \int 3x^3 \sqrt{x^4 + 12} \, dx = \int 3x^3 \, \frac{u^2}{4x} \, du \]
\[= \frac{1}{4} \int u^2 \, du \]
\[= \frac{1}{4} \left[ \frac{4}{3} u^\frac{3}{2} + C \right] \]
\[= \frac{1}{6} (x^4 + 12)^{\frac{3}{2}} + C \]
\[ f(x) = x^3 + x \quad y = 0 \quad x = 8 \]

\[ w = \frac{8 - 0}{4} = 2 \]

\[ S_y = w \left[ f(2) + f(4) + f(6) + f(8) \right] \]

\[ S_y = 2 \left[ 10 + 68 + 222 + 520 \right] \]

\[ S_y = 1640 \]
Compute $A_I$:  

$$A_I = \int_{-5\sqrt{10}}^{0} \left[ x^3 - 9x - x \right] \, dx$$

$$= \int_{-5\sqrt{10}}^{0} \left[ x^3 - 10x \right] \, dx$$

$$= \left[ \frac{1}{4} x^4 - 5x^2 \right]_{-5\sqrt{10}}^{0}$$

$$= 0 - \left[ \frac{1}{4} \cdot (5\sqrt{10})^4 - 5(5\sqrt{10})^2 \right]$$

$$= 0 - \left[ \frac{1}{4} \cdot 1000 - 5(10) \right]$$

$$= 0 - (25 - 50)$$

$$A_I = 25.$$

Compute $A_{II}$:  

$$A_{II} = \int_{0}^{5\sqrt{10}} x - \left( x^3 - 9x \right) \, dx$$

$$= \int_{0}^{5\sqrt{10}} \left( x - x^3 + 9x \right) \, dx$$

$$= \int_{0}^{5\sqrt{10}} \left( 10x - x^3 \right) \, dx$$

$$= \left[ 5x^2 - \frac{1}{4} x^4 \right]_{0}^{5\sqrt{10}}$$

$$= 5(5\sqrt{10})^2 - \frac{1}{4} (5\sqrt{10})^4$$

$$= 25.$$

Total Area $= A_I + A_{II}$

$= 25 + 25$

$= 50$
\[ p = 500 - q^2 \quad \text{Find CS.} \]
\[ p = 30q + 100 \]

\[ p = 500 - q^2 \]
\[ p = 30q + 100 \]
\[ p = 400 \]

\[ 500 - q^2 = 30q + 100 \]
\[ 0 = q^2 + 30q - 400 \]
\[ 0 = (q + 40)(q - 10) \]
\[ q = 10 \quad \text{or} \quad q = 40 \]

To find \( p_0 \): use either the demand or supply.
Using demand: \[ p = 500 - (10)^2 = 500 - 100 = 400 \]

CS is the area between the top curve and the eq. line:

\[ CS = \int_{0}^{10} (\text{top} - \text{bot}) \, dq \]
\[ = \int_{0}^{10} (500 - q^2) - 100 \, dq \]
\[ = \int_{0}^{10} (100 - q^2) \, dq \]
\[ = 100q - \frac{1}{3}q^3 \bigg|_0^{10} \]
\[ = \left[ 100(10) - \frac{1}{3}(10)^3 \right] - 0 \]
\[ = 1000 - \frac{1}{3}(1000) \]
\[ = \frac{2000}{3} \quad \text{or} \quad 666.67 \]