

# Math 1131 Review for Midterm 1

1. Find the following limits:

a.  $\lim_{x \rightarrow 7^+} \frac{x^2 - 4x - 21}{x^2 - 2x - 35}$

b.  $\lim_{h \rightarrow 0} \frac{\frac{3}{x+h} - \frac{3}{x}}{h}$

c.  $\lim_{t \rightarrow -2^+} \frac{7-t}{t^2 - t - 6}$

d.  $\lim_{x \rightarrow -\infty} \frac{x^3 - 3x^5 - 3x^6}{4x^6 - 7}$

e.  $\lim_{x \rightarrow 0} \frac{\sqrt{4+12x} - 2}{x}$

2. Let  $h(x)$  be given by the following graph. Find:

a.  $h(1)$

h.  $\lim_{x \rightarrow 1^-} h(x)$

b.  $h(-1)$

i.  $\lim_{x \rightarrow 1} h(x)$

c. Domain of  $h$

d. Range of  $h$

e.  $\lim_{x \rightarrow -1} h(x)$

j.  $\lim_{x \rightarrow 2} h(x)$

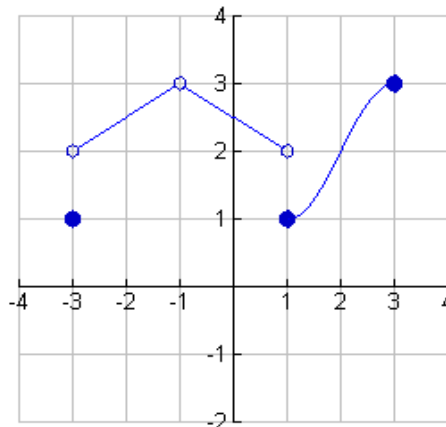
f.  $\lim_{x \rightarrow -3^+} h(x)$

k. Is  $h(x)$  continuous at -1?

g.  $\lim_{x \rightarrow 1^+} h(x)$

l. Is  $h(x)$  continuous at 1?

m. Is  $h(x)$  continuous at 2?



2. Let  $f(x) = \begin{cases} \frac{12}{(x-3)(x+5)} & x < -1 \\ x+2 & -1 \leq x \leq 3 \\ x^2 - 4 & x > 3 \end{cases}$

Find:

a.  $f(-1)$

b.  $f(3)$

c.  $\lim_{x \rightarrow -1} f(x)$

d.  $\lim_{x \rightarrow 3^-} f(x)$

e.  $\lim_{x \rightarrow 3^+} f(x)$

f. List all discontinuities of this function.

4. Solve the inequality using a sign chart:  $\frac{(x-3)(x+2)}{x+4} \leq 0$

5. Determine where the function  $f(x) = \frac{x^2 + 5}{x^2 + 3x + 2}$  is continuous.

6. Use the definition of derivative to find  $f'(x)$  if  $f(x) = x^2 + 3x$ .

7. Use the differentiation rules to find the derivative of the following:

a.  $y = 2x^5 - \sqrt{x} + \frac{3}{x^2}$

b.  $y = (x^3 - 4)^2$

8. Find an equation of the tangent line to the curve  $y = x^3 + 2x - 5$  when  $x=2$ .

9. A manufacturer's revenue function is given by  $r(q) = 250q - 10q^2 - q^3$ .

a. Find the marginal revenue function

b. Find the marginal revenue when 5 units are produced.

10. The total profit from selling  $x$  units of gardening shovels is  $P(x) = (6x - 7)(9x - 8)$ . Find the marginal average profit function.

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11. Find the derivative of each of the following. Do not simplify.

a.  $y = (x^3 - 7x^2 + 5x - 3)^9$

b.  $f(x) = \sqrt[4]{x^3 - 7x}$

c.  $y = \frac{\sqrt[7]{x^2 - 9x}}{\sqrt{x^{3/4} - 5}}$

d.  $f(t) = (t^2 + 4)^5 (t^3 - 9)^6$

e.  $y = \left(\frac{x+7}{x^2-3}\right)^5$

f.  $y = \sqrt[3]{7} + e^\pi + 7x$

g.  $g(x) = e^{x^2-4x+3} + \frac{1}{e^{\sqrt{2x-3}}}$

h.  $y = x^3 e^x$

## Answers

1. a.  $\lim_{x \rightarrow 7^+} \frac{x^2 - 4x - 21}{x^2 - 2x - 35} = \frac{5}{6}$

b.  $\lim_{h \rightarrow 0} \frac{\frac{3}{x+h} - \frac{3}{x}}{h} = \frac{-3}{x^2}$

c.  $\lim_{t \rightarrow -2^+} \frac{7-t}{t^2-t-6} = -\infty$

d.  $\lim_{x \rightarrow -\infty} \frac{x^3 - 3x^5 - 3x^6}{4x^6 - 7} = \frac{-3}{4}$

e.  $\lim_{x \rightarrow 0} \frac{\sqrt{4+12x} - 2}{x} = 3$

2. a.  $h(1) = 1$

b.  $h(-1)$  DNE

c.  $[-3, -1) \cup (-1, 3]$

d.  $[1, 3]$

e.  $\lim_{x \rightarrow -1} h(x) = 3$

f.  $\lim_{x \rightarrow -3^+} h(x) = 2$

g.  $\lim_{x \rightarrow 1^+} h(x) = 1$

h.  $\lim_{x \rightarrow 1^-} h(x) = 2$

i.  $\lim_{x \rightarrow 1} h(x)$  DNE

j.  $\lim_{x \rightarrow 2} h(x) = 2$

k. no

l. no

m. yes

3. a.  $f(-1) = 1$

b.  $f(3) = 5$

c.  $\lim_{x \rightarrow -1} f(x) = \text{DNE}$

d.  $\lim_{x \rightarrow 3^-} f(x) = 5$

e.  $\lim_{x \rightarrow 3^+} f(x) = 5$

f.  $x = -1, x = -5$

4.  $(-\infty, -4) \cup [-2, 3]$

5.  $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$

6.  $f'(x) = 2x + 3$

7. a.  $\frac{dy}{dx} = 10x^4 - \frac{1}{2\sqrt{x}} - \frac{6}{x^3}$

b.  $\frac{dy}{dx} = 6x^5 - 24x^2$

8.  $y = 14x - 21$

9. a.  $r'(q) = 250 - 20q - 3q^2$

b.  $r'(5) = 75$

10.  $\bar{P}(x) = 54 - \frac{56}{x^2}$

11. a.  $y' = 9(x^3 - 7x^2 + 5x - 3)^8 (3x^2 - 14x + 5)$

b.  $f'(x) = \frac{1}{4}(x^3 - 7x)^{-3/4} (3x^2 - 7)$

c.  $y' = \frac{\left(\frac{2}{7}x^{-2/7} - 9\right)(x^{3/4} - 5)^{1/2} - \left(\frac{3(\sqrt[7]{x^2 - 9x})}{8x^{1/4}\sqrt{x^{3/4} - 5}}\right)}{(x^{3/4} - 5)}$

d.  $f'(t) = [5(t^2 + 4)^4 (2t)](t^3 - 9)^6 + (t^2 + 4)^5 [6(t^3 - 9)^5 (3t^2)]$

e.  $y' = 5\left(\frac{x+7}{x^2-3}\right)^4 \left(\frac{(x^2-3) - (x+7)(2x)}{(x^2-3)^2}\right)$

f.  $y'(x) = 7$

g.  $g'(x) = e^{x^2-4x+3} (2x-4) + e^{-\sqrt{2x-3}} \left(-\frac{1}{2}(2x-3)^{-1/2} \cdot 2\right)$

h.  $y' = 3x^2 e^x + x^3 e^x$