

MSLC – Math 1148
Final Exam Review

Disclaimer: This should NOT be used as your only guide for what to study.

1. Use the piece-wise defined function $f(x) = \begin{cases} x^2 + 4x + 4 & \text{if } x < 0 \\ -\frac{1}{2}x + 2 & \text{if } x \geq 2 \end{cases}$ to answer the following:

- Compute $f(-3)$, $f(-2)$, $f(-1)$, $f(2)$, and $f(4)$.
- Plot the points you found above and sketch a complete graph of $y = f(x)$.
- Find the domain of f . Write your answer using interval notation.

2. Use your graphing calculator to graph each of the functions below over the interval $(-2, 2)$ and approximate any local extrema. Also, determine the intervals where the functions are increasing and decreasing. Round your answers to three decimal places where appropriate.

- $f(x) = x^2$
- $g(x) = (x-1)^2(x+1)^2$
- $h(x) = \sqrt{|x|}$
- $k(x) = x(x-1)(x+1)$

3. Determine the average rate of change of the functions between the given values of x .

- $h(x) = 3x - 17$ from $x = -1$ to $x = 2$
- $f(x) = \frac{2x}{3x+1}$ from $x = 1$ to $x = t$
- $g(x) = \frac{1}{\sqrt{x+1}}$ from $x = 0$ to $x = a$

4. Write the equation of the function $F(x) = \sqrt{x}$ transformed in the following ways:

- shifted 2 units to the left, and shifted up 3 units
- reflected about the x -axis, then shifted down 3 units
- shifted 1 unit to the right, and vertically stretched by a factor of 3
- shifted 1 unit to the left, then reflected about the y -axis

5. Given $f(x) = \sqrt[3]{x+5}$ and $g(x) = \frac{1}{4x+1}$ find:

- $f \circ g$
- $g \circ f$
- $f \circ f$
- $g \circ g$

6. Find the inverse of each function, state the domain and range of both the original function and its inverse.

a) $f(x) = 2x - 1$

b) $g(x) = x^3 + 1$

c) $k(x) = \frac{1}{x-3}$

d) $h(x) = \frac{x}{5-2x}$

7. Solve the following inequalities. Write your answer using interval notation.

a) $5(1-2x) \leq 9(x-3)$

b) $\frac{2}{3} - \frac{1}{2}x \geq \frac{1}{6} + x$

c) $(x+3)^2(x+1) < 0$

d) $\frac{x}{x+1} > 3x$

e) $|4x+1| < 17$

f) $|2x-1| \geq 5$

8. Find all real zeros and the multiplicity of those zeros for the given polynomials, then find their y-intercepts and sketch the graph.

a) $p(x) = (2x-1)(x+1)(x+3)$

b) $p(x) = \frac{1}{4}(x+1)^3(x-3)$

c) $p(x) = (x+1)^2(x-3)^2$

9. Factor each polynomial, then use the factored form to find the zeros and the multiplicity of each zero.

a) $P(x) = x^3 + 2x^2 - 8x$

b) $P(x) = x^5 - 9x^3$

c) $P(x) = x^3 + 3x^2 - 4x - 12$

10. Determine whether or not each equation is a function of x .

a) $y = x^2 - 3x + 4$

b) $x^2 + y^2 = 1$

c) $x^2 + y^3 = 4$

d) $xy = 1$

11. Find the domain of each of the following functions. *Give your answer in interval notation.*

a) $f(x) = \frac{(x-5)(x+95)}{8(8x+5)(x-3)}$

b) $g(x) = \sqrt{3(x-11)}$

c) $h(x) = \frac{x^2 - 4}{x^2 - 10x - 24}$

d) $j(x) = \sqrt{3-2x}$

e) $k(x) = \sqrt[3]{x-7}$

12. Find all intercepts and asymptotes, then sketch the graph of each rational function.

a. $R(x) = \frac{x-2}{x^2-4x}$

b. $R(x) = \frac{x^2+3x}{x^2-x-6}$

c. $R(x) = \frac{x^3+4}{2x^2+x-1}$

13. Solve the equation.

a. $2^{2w+1} = 4^{2w-1}$

b. $27^{x+9} = 81^x$

c. $2 \ln(r) = \ln(15r + 34)$

d. $\log_4(2x-1) = 2$

e. $\log_3(x^2 + 24x) = 4$

f. $e^{-2x} = 4$

g. $\log_6(x^2 + 6x + 41) = 2$

h. $4 \cdot 5^{2x+1} = 3^{x+3}$

i. $3^{2x} - 3^{x+1} = -2$

14. How much will be in an account after five years if \$2000 is deposited into the account that yields 6% compounded:

a) semiannually?

b) quarterly?

c) monthly?

d) daily?

e) continuously?

15. Determine (correct to 3 decimal places) how long it will take for \$2000 to double if it's invested in an account that gives 6% interest compounded:

a) semiannually?

b) quarterly?

c) monthly?

d) daily?

e) continuously?

16. Find the quotient and remainder of the following:

a. $\frac{x^2 + 4x - 9}{x + 3}$

b. $\frac{x^3 + 6x + 5}{x^2 - 2x + 2}$

c. $\frac{2x^3 + 7x^2 + 6x - 5}{2x - 1}$

17. A boat on a river travels downstream for 20 miles in one hour. The return trip back upstream takes 2.5 hours. How fast does the boat travel in still water, and how fast is the current?

18. A fruit stand sells a box of Strawberries for \$7 and a box of Kiwi fruit for \$10. If they sold a total of 135 boxes of fruit and had revenue of \$1110, how many boxes of each fruit did they sell?

19. Solve the following system of linear equations:

a) $\begin{cases} 3x + 2y = 0 \\ -x - 2y = 8 \end{cases}$

b) $\begin{cases} 15x - 6y = 9 \\ 2y - 5x = 14 \end{cases}$

c) $\begin{cases} x - 2y + 3z = 1 \\ x + 2y - z = 13 \\ 3x + 2y - 5z = 3 \end{cases}$

20. Write the equation of the line having the given properties:

a) point-slope form; passing through the points (2, 7) and (-1, -2)

b) slope-intercept form; passing through (6, -1) and perpendicular to $4x - 3y = -21$

c) no slope; passing through the point (8, 13)

21. A company that makes widgets has determined that its monthly profit can be approximately by the function $f(p) = -80p^2 + 3440p - 36,000$ where p is the price per widget and $f(p)$ is the monthly profit based on that price.

a. What price(s) allows the company to break even?

b. Find the price that generates the maximum profit.

c. Find the maximum profit.

22. A certain radioactive isotope decays according to the function $m(t) = 12e^{-0.015t}$ where m is the mass of the isotope remaining measured in kilograms after t days.

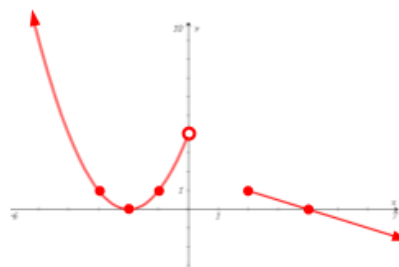
a. How much of the isotope is present initially?

b. How much of the isotope remains after 45 days? *Round to 2 decimal places.*

Answers

1. a) $f(-3)=1, f(-2)=0, f(-1)=1, f(2)=1, f(4)=0$ b)

c) $(-\infty, 0) \cup [2, \infty)$



2. a) Min: $(0, 0)$; Increasing: $(0, 2)$; Decreasing: $(-2, 0)$

b) Mins: $(-1, 0)$ & $(1, 0)$; Max: $(0, 1)$ Increasing: $(-1, 0) \cup (1, 2)$; Decreasing: $(-2, -1) \cup (0, 1)$

c) Min: $(0, 0)$; Increasing: $(0, 2)$; Decreasing: $(-2, 0)$

d) Min: $(0.578, -0.385)$; Max: $(-0.578, 0.385)$;

Increasing: $(-2, -0.578) \cup (0.578, 2)$; Decreasing: $(-0.578, 0.578)$

3. a) 3 b) $\frac{1}{6t+2}$ c) $\frac{1-\sqrt{a+1}}{a\sqrt{a+1}}$

4. a) $y = \sqrt{x+2} + 3$ b) $y = -\sqrt{x} - 3$ c) $y = 3\sqrt{x-1}$ d) $y = \sqrt{-(x-1)}$

5. a) $(f \circ g)(x) = \sqrt[3]{\frac{20x+6}{4x+1}}$ b) $(g \circ f)(x) = \frac{1}{4\sqrt[3]{x+5} + 1}$ c) $(f \circ f)(x) = \sqrt[3]{\sqrt[3]{x+5} + 5}$ d) $(g \circ g)(x) = \frac{4x+1}{4x+5}$

6. a) $f^{-1}(x) = \frac{x+1}{2}$ Domain $f(x) = (-\infty, \infty)$; Range $f(x) = (-\infty, \infty)$

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b) $g^{-1}(x) = \sqrt[3]{x-1}$ Domain $g(x) = (-\infty, \infty)$; Range $g(x) = (-\infty, \infty)$

Domain $g^{-1}(x) = (-\infty, \infty)$; Range $g^{-1}(x) = (-\infty, \infty)$

c) $k^{-1}(x) = \frac{1}{x} + 3$ Domain $k(x) = (-\infty, 3) \cup (3, \infty)$; Range $k(x) = (-\infty, 0) \cup (0, \infty)$

Domain $k^{-1}(x) = (-\infty, 0) \cup (0, \infty)$; Range $k^{-1}(x) = (-\infty, 3) \cup (3, \infty)$

d) $h^{-1}(x) = \frac{5x}{1+2x}$ Domain $h(x) = \left(-\infty, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$; Range $h(x) = \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$

Domain $h^{-1}(x) = \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$; Range $h^{-1}(x) = \left(-\infty, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$

7. a) $\left[\frac{32}{19}, \infty\right)$ b) $\left(-\infty, \frac{1}{3}\right]$ c) $(-\infty, -3) \cup (-3, -1)$ d) $(-\infty, -1) \cup \left(-\frac{2}{3}, 0\right)$ e) $\left(-\frac{9}{2}, 4\right)$

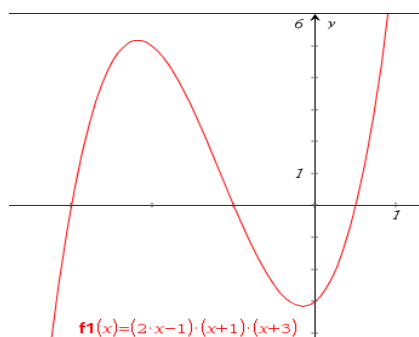
f) $(-\infty, -2] \cup [3, \infty)$

8. a) Zeros: $\frac{1}{2}$ (multiplicity: 1);

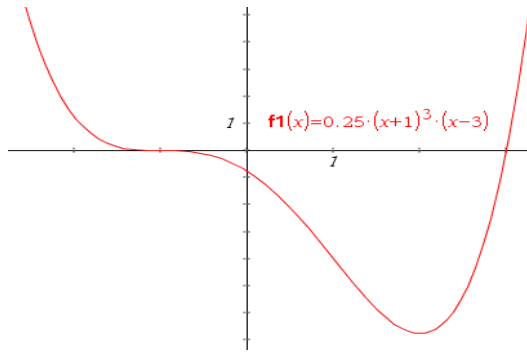
-1 (multiplicity: 1);

-3 (multiplicity: 1);

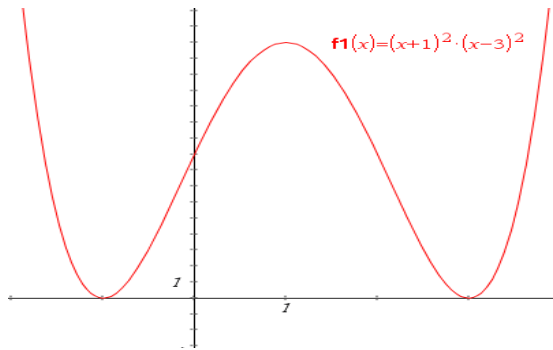
y-int: $(0, -3)$



8. b) Zeros: -1 (multiplicity:3);
 3 (multiplicity:1);
 y -int: $(0, -\frac{3}{4})$



- c) Zeros: -1 (multiplicity:2);
 3 (multiplicity:2);
 y -int: $(0, 9)$

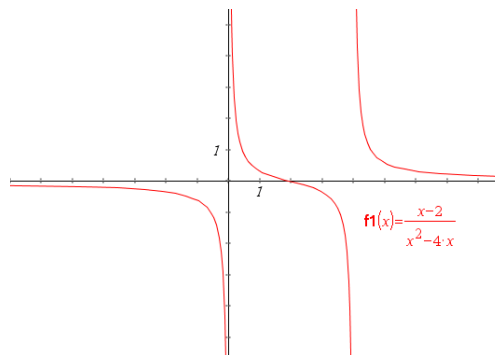


9. a) $P(x) = x(x+4)(x-2)$; Zeros: 0 (multiplicity:1); -4 (multiplicity:1); 2 (multiplicity:1)
 b) $P(x) = x^3(x+3)(x-3)$; Zeros: 0 (multiplicity:3); -3 (multiplicity:1); 3 (multiplicity:1)
 c) $P(x) = (x+3)(x-2)(x+2)$; Zeros: 2 (multiplicity:1); -2 (multiplicity:1); -3 (multiplicity:1)

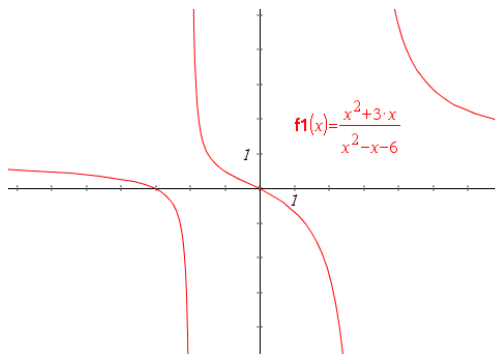
10. a) Function b) Not a function c) Function d) Function

2. 11. a) $(-\infty, -\frac{5}{8}) \cup (-\frac{5}{8}, 3) \cup (3, \infty)$ b. $[11, \infty)$ c. $(-\infty, -2) \cup (-2, 12) \cup (12, \infty)$
 d. $(-\infty, \frac{3}{2}]$ e. $(-\infty, \infty)$

12. a) no y -intercept,
 x -intercept: $(2, 0)$;
 Horizontal Asymptote: $y = 0$;
 Vertical Asymptotes: $x = 0$; $x = 4$



- b) y -intercept: $(0, 0)$
 x -intercepts: $(0, 0)$ & $(-3, 0)$
 Horizontal Asymptote: $y = 1$
 Vertical Asymptotes: $x = -2$; $x = 3$

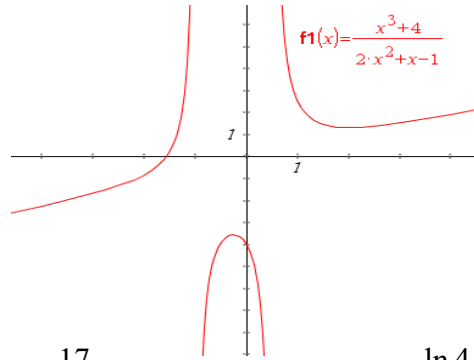


12.c) y-intercept: $(0, -4)$

x-intercepts: $(\sqrt[3]{-4}, 0)$

Slant Asymptote: $y = \frac{1}{2}x - \frac{1}{4}$;

Vertical Asymptotes: $x = -1$; $x = \frac{1}{2}$



13. a. $w = \frac{3}{2}$ b. $x = 27$ c. $r = 17$ d. $x = \frac{17}{2}$ e. $x = 3$ f. $x = -\frac{\ln 4}{2}$

g. $x = -5$ or $x = -1$ h. $x = \frac{\ln 4 + \ln 5 - 3 \ln 3}{\ln 3 - 2 \ln 5}$ i. $x = \frac{\ln 2}{\ln 3}$ or $x = 0$

14. a) \$2687.83 b) \$2693.71 c) \$2697.70 d) 2699.65 e) 2699.72

15. a) 11.725 yrs b) 11.639 yrs c) 11.581 yrs d) 11.553 yrs e) 11.552 yrs

16. a) $x + 1 - \frac{12}{x + 3}$; b) $x + 2 + \frac{8x + 1}{x^2 - 2x + 2}$; c) $x^2 + 4x + 5$

17. Boat's speed in still water = 14mph; Current's speed = 6mph

18. 80 boxes of Strawberries; 55 boxes of Kiwi fruit

19. a) $(4, -6)$ b) No solution c) $(3, 7, 4)$

20. a) $y - 7 = 3(x - 2)$ or $y + 2 = 3(x + 1)$ b) $y = -\frac{3}{4}x + \frac{7}{2}$ c) $x = 8$

21. a) prices: \$18 and \$25 b) price: \$21.50 c) Max. profit: \$980

22. a) 12 kg; b. 6.11 kg