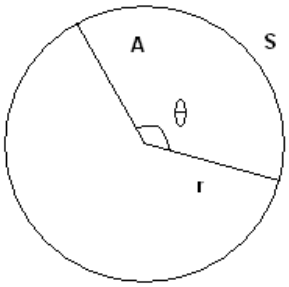
MSLC – Math 1149
Final Exam Review

Disclaimer: This should NOT be used as your only guide for what to study.

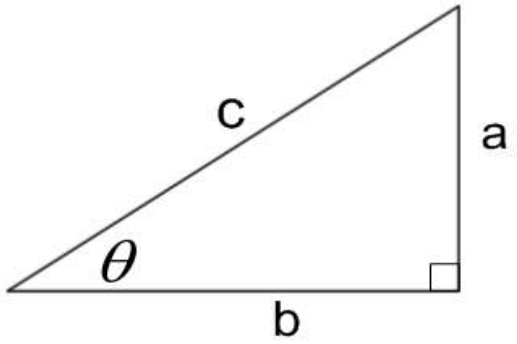
1. In the circle pictured below, r is the radius of the circle, θ is the central angle of the sector, A is the area of the sector, and s is the length of the arc subtended by the central angle. Find:



   a. A and s if $r = 3$ inches, and $\theta = \frac{\pi}{3}$ radians
   b. r and s if $A = 2.25$ square miles, and $\theta = 36°$
   c. A and θ if $r = 4$ meters, and $s = 12.57$ meters
   d. r and A if $s = 13$ meters, and $\theta = \frac{\pi}{4}$ radians

2. Use the given information to find the exact value of the six trigonometric functions of the angle θ in the picture.

   a) $a = 8,\ b = 15$
   b) $a = 12, c = 13$
   c) $\sin\theta = \frac{2}{7}$
   d) $\cot\theta = 3$
   e) $\cos\theta = \frac{3}{5}$
   f) $\csc\theta = \sqrt{5}$



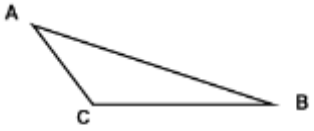
3. For each part, determine the quadrant θ lies in and the values of the five remaining trigonometric functions.

   a) $\csc\theta = -\sqrt{65}$ and $\cot\theta > 0$
   b) $\sec\theta = -\frac{13}{4\sqrt{3}}$ and $\sin\theta > 0$
   c) $\tan\theta = -\frac{35}{12}$ and $\cos\theta > 0$

4. Find the exact value of each of the following.

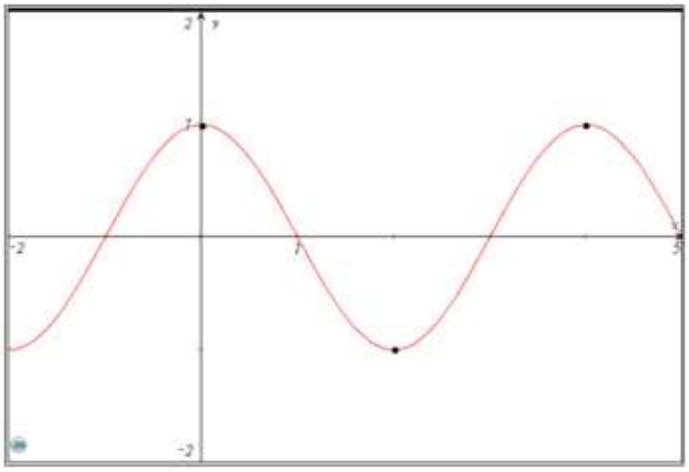
   a. $\tan\left(\sin^{-1}\left(-\frac{99}{101}\right)\right)$
   b. $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right)$

5. Given triangle $\triangle ABC$ with the following properties:
   $b = 14,\ c = 17,\ B = 44°$ and C is an obtuse angle
   Find the measure of angle C.
   Round your answer to 2 decimal places.

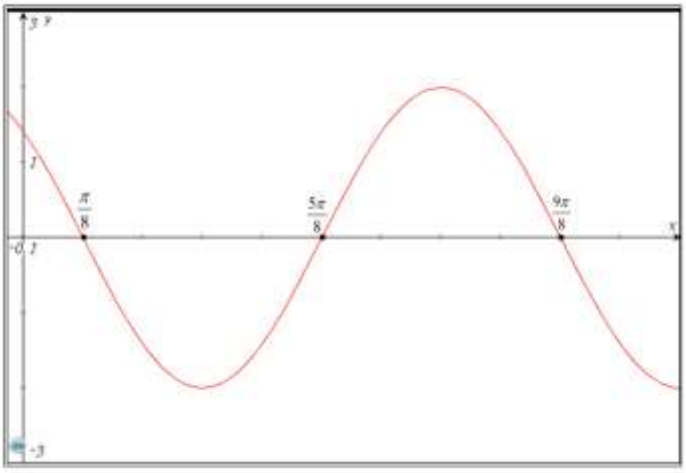


6. Given the following graph, find an equation that will satisfy the given conditions.

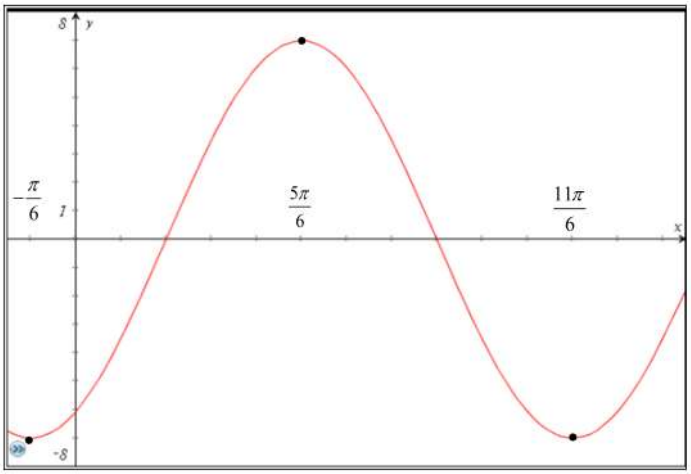
a. $y = a\cos k(x-b)$



b. $y = a\sin k(x-b)$



c. $y = a\cos k(x-b)$



7. Find the period, amplitude, and phase-shift of the given trigonometric equation, then graph one complete period.

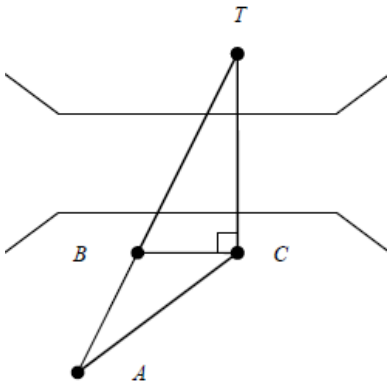
a. $y = 3\sin(\pi x)$

b. $y = 2\cos 4\left(x + \frac{\pi}{6}\right)$

c. $y = -5\sin\left(2x + \frac{\pi}{4}\right)$

8. A team of surveyors have been hired to measure the distance across a canyon. Using a tree at point *T* on the opposite site of the canyon as a reference point, they established points *A*, *B*, and *C* and found the following distances:

AB = 12.25 ft
BC = 6.5 ft
AC = 15 ft



a. Find the measure of angle *ABC*. *Round your answer to 2 decimal places*.

b. Find the distance *TC* across the canyon to the nearest foot.

9. Establish the identity:

a. $\frac{1-\sin x}{1+\sin x} = (\sec x - \tan x)^2$

b. $\frac{\sec\theta + \csc\theta}{\tan\theta + \cot\theta} = \sin\theta + \cos\theta$

c. $\sin\left(\frac{\pi}{2} - x\right) = \sin\left(\frac{\pi}{2} + x\right)$

d. $\frac{1+\sin 2x}{\sin 2x} = 1 + \frac{1}{2}\sec x \csc x$

10. Find the exact solution of:

a. $\sin\frac{11\pi}{12}$

b. tan(165°)

c. $\cos\frac{7\pi}{8}$

11. Find all solutions of:

a. $\sqrt{2}\cos 2x + 1 = 0$ on the interval $[0, 2\pi)$

b. $\cos x \sin x - 2\cos x = 0$

c. $2\sin\frac{x}{2} - \sqrt{3} = 0$ on the interval $[0, 8\pi)$

12. Given $\cot x = -\frac{2}{3}$ and $\sin x > 0$ find:

a. $\sin 2x$

b. $\sin \frac{x}{2}$

c. $\cos 2x$

d. $\cos \frac{x}{2}$

13. Given the complex numbers $z_1 = -\frac{3}{2} - \frac{3\sqrt{3}}{2}i$ and $z_2 = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

a. Convert both z_1 and z_2 to polar coordinates

b. Find $z_1 z_2$

c. Find $\frac{z_1}{z_2}$

d. Find $\frac{1}{z_1}$

14. Use DeMoivre's Theorem to find z^7 in the standard $a + bi$ form:

a. $z = 1 + i$

b. $z = 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

15. Given $\mathbf{u} = 2\mathbf{i} + \mathbf{j}$ and $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j}$ find:

a. $\mathbf{u} + \mathbf{v}$

b. $3\mathbf{u} - 4\mathbf{v}$

c. $|\mathbf{u} + \mathbf{v}|$

d. $|3\mathbf{u} - 4\mathbf{v}|$

e. $\mathbf{u} \cdot \mathbf{v}$

f. the angle (in degrees) between **u** and **v.** *Round your answer to 3 decimal places*.

16. a. Find the equation of the parabola with vertex at the origin and a directrix of $x = -6$

b. Graph the parabola, making sure to plot and label all information given in part a.

17. a. Find the equation of an ellipse with eccentricity of $\frac{1}{9}$ and foci at $(0, \pm 2)$

b. Graph the ellipse, making sure to plot and label all information given in part a.

18. a. Find the transverse axis, vertices, foci, and the equations of the asymptotes of the hyperbola described by the equation $9x^2 - 4y^2 = 36$.

b. Graph the hyperbola, making sure to plot and label all information given in part a.

ANSWERS

1. a. $s = 3.142$ inches $A = 4.712$ in^2 b. $r = 2.676$ miles $s = 1.681$ miles

c. $\theta = \pi$ (radians) $A = 25.13$ m^2 d. $r = 16.552$ meters $A = 107.589$ m^2

2. a. $\sin\theta = \frac{8}{17}$ $\cos\theta = \frac{15}{17}$ $\tan\theta = \frac{8}{15}$ $\csc\theta = \frac{17}{8}$ $\sec\theta = \frac{17}{15}$ $\cot\theta = \frac{15}{8}$

b. $\sin\theta = \frac{12}{13}$ $\cos\theta = \frac{5}{13}$ $\tan\theta = \frac{12}{5}$ $\csc\theta = \frac{13}{12}$ $\sec\theta = \frac{13}{5}$ $\cot\theta = \frac{5}{12}$

c. $\sin\theta = \frac{2}{7}$ $\cos\theta = \frac{\sqrt{45}}{7}$ $\tan\theta = \frac{2}{\sqrt{45}}$ $\csc\theta = \frac{7}{2}$ $\sec\theta = \frac{7}{\sqrt{45}}$ $\cot\theta = \frac{\sqrt{45}}{2}$

d. $\sin\theta = \frac{1}{\sqrt{10}}$ $\cos\theta = \frac{3}{\sqrt{10}}$ $\tan\theta = \frac{1}{3}$ $\csc\theta = \sqrt{10}$ $\sec\theta = \frac{\sqrt{10}}{3}$ $\cot\theta = 3$

e. $\sin\theta = \frac{4}{5}$ $\cos\theta = \frac{3}{5}$ $\tan\theta = \frac{4}{3}$ $\csc\theta = \frac{5}{4}$ $\sec\theta = \frac{5}{3}$ $\cot\theta = \frac{3}{4}$

f. $\sin\theta = \frac{1}{\sqrt{5}}$ $\cos\theta = \frac{2}{\sqrt{5}}$ $\tan\theta = \frac{1}{2}$ $\csc\theta = \sqrt{5}$ $\sec\theta = \frac{\sqrt{5}}{2}$ $\cot\theta = 2$

3. a. θ is in Quadrant III $\sin\theta = -\frac{1}{\sqrt{65}}$; $\cos\theta = -\frac{8}{\sqrt{65}}$; $\tan\theta = \frac{1}{8}$

$\csc\theta = -\sqrt{65}$ $\sec\theta = -\frac{\sqrt{65}}{8}$ $\cot\theta = 8$

b. θ is in Quadrant II $\sin\theta = \frac{11}{13}$ $\cos\theta = -\frac{\sqrt{48}}{13}$ $\tan\theta = -\frac{11}{\sqrt{48}}$

$\csc\theta = \frac{13}{11}$ $\sec\theta = -\frac{13}{\sqrt{48}}$ $\cot\theta = -\frac{\sqrt{48}}{11}$

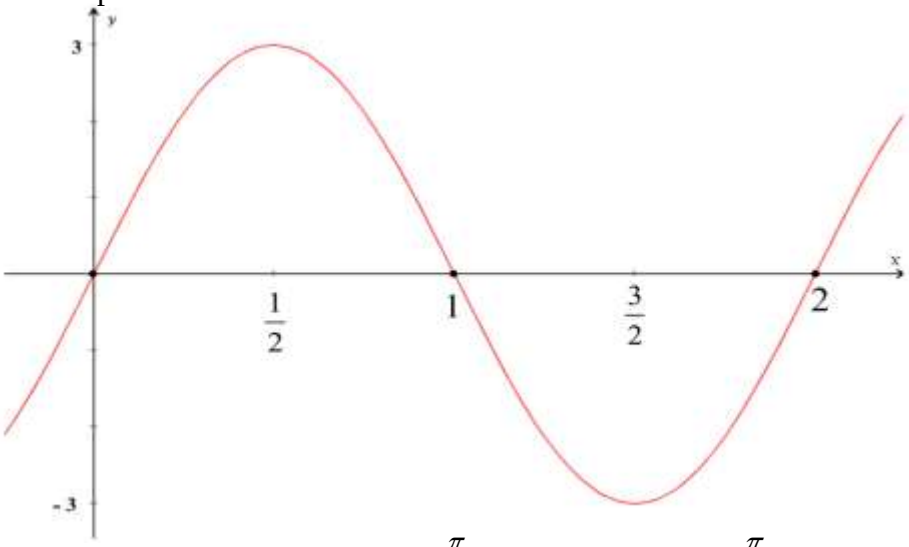
c. θ is in Quadrant IV $\sin\theta = -\frac{35}{37}$ $\cos\theta = \frac{12}{37}$ $\tan\theta = -\frac{35}{12}$

$\csc\theta = -\frac{37}{35}$ $\sec\theta = \frac{37}{12}$ $\cot\theta = -\frac{12}{35}$

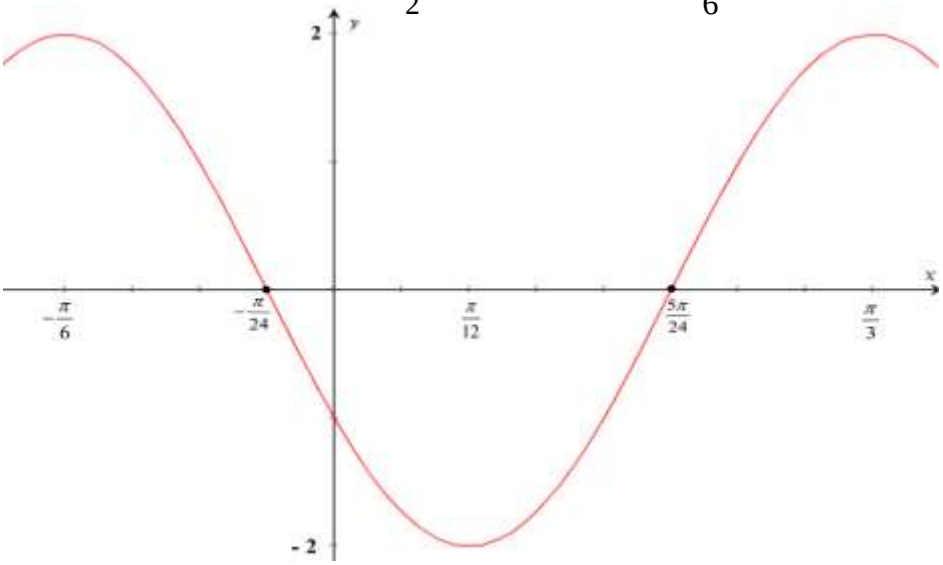
4. a. $-\frac{99}{20}$ b. $\frac{2\pi}{3}$ 5. $C = 122.49^\circ$

6. a. $f(x) = \cos\left(\frac{\pi}{2}x\right)$ b. $g(x) = -2\sin 2\left(x - \frac{\pi}{8}\right)$ c. $h(x) = -7\cos\left(x + \frac{\pi}{6}\right)$

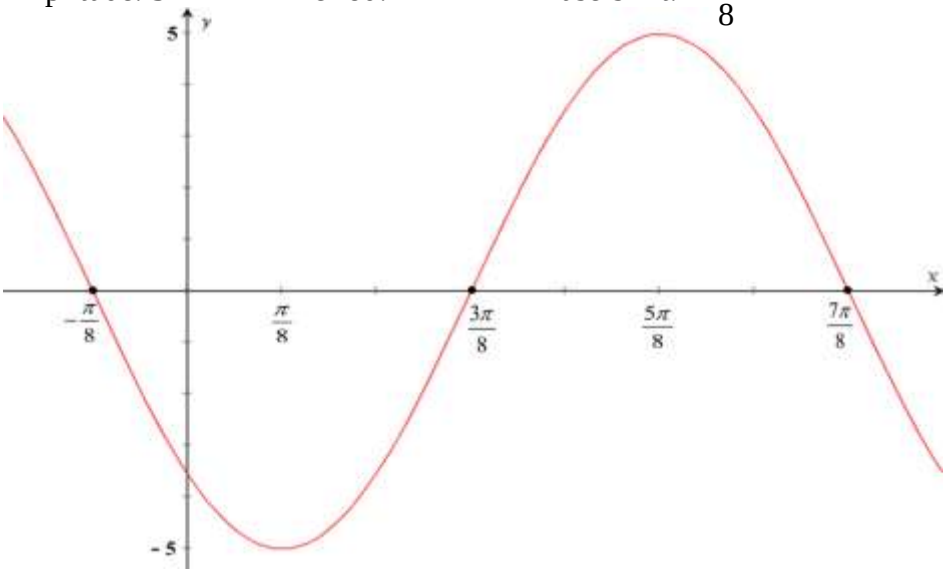
7. a. Amplitude: 3 Period: 2 Phase-shift: none



b. Amplitude: 2 Period: $\frac{\pi}{2}$ Phase-shift: $-\frac{\pi}{6}$



c. Amplitude: 5 Period: π Phase-shift: $-\frac{\pi}{8}$



8. a. $\angle ABC = 101.84°$ b. $TC = 31$ feet

9. a.
$$\frac{1-\sin x}{1+\sin x} = (\sec x - \tan x)^2$$
$$\frac{1-\sin x}{1+\sin x}\cdot\frac{1-\sin x}{1-\sin x} = RHS$$
$$\frac{1-2\sin x+\sin^2 x}{1-\sin^2 x} = RHS$$
$$\frac{1-2\sin x+\sin^2 x}{\cos^2 x} = RHS$$
$$\frac{1}{\cos^2 x}-\frac{2\sin x}{\cos^2 x}+\frac{\sin^2 x}{\cos^2 x} = RHS$$
$$\sec^2 x - 2\sec x\tan x + \tan^2 x = (\sec x - \tan x)^2$$

b.
$$\frac{\sec x+\csc x}{\tan x+\cot x} = \sin x+\cos x$$
$$\frac{\frac{1}{\cos x}+\frac{1}{\sin x}}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}} = RHS$$
$$\frac{\frac{\sin x+\cos x}{\cos x\sin x}}{\frac{\sin^2 x+\cos^2 x}{\cos x\sin x}} = RHS$$
$$\frac{\sin x+\cos x}{\cos x\sin x}\cdot\frac{\cos x\sin x}{\sin^2 x+\cos^2 x} = \sin x+\cos x$$

$$\sin\left(\frac{\pi}{2}-x\right) = \sin\left(\frac{\pi}{2}+x\right)$$
$$LHS = \sin\left(\frac{\pi}{2}\right)\cos(x)+\sin x\cos\left(\frac{\pi}{2}\right)$$

c. $LHS = 1\cdot\cos x + 0$
$LHS = \cos x$
$$\sin\left(\frac{\pi}{2}-x\right) = \cos x$$

d.
$$\frac{1+\sin 2x}{\sin 2x} = 1+\frac{1}{2}\sec x\csc x$$
$$\frac{\sin 2x}{\sin 2x}+\frac{1}{\sin 2x} = RHS$$
$$1+\frac{1}{2\sin x\cos x} = RHS$$
$$1+\frac{1}{2}\csc x\sec x = 1+\frac{1}{2}\sec x\csc x$$

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10. a. $\frac{\sqrt{6}-\sqrt{2}}{4}$ b. $\frac{1-\sqrt{3}}{1+\sqrt{3}}$ c. $-\sqrt{\frac{2+\sqrt{2}}{4}}$

11. a. $\frac{3\pi}{8},\frac{5\pi}{8},\frac{11\pi}{8},\frac{13\pi}{8}$ b. $\frac{\pi}{2}+\pi k$ c. $\frac{2\pi}{3},\frac{4\pi}{3},\frac{14\pi}{3},\frac{16\pi}{3}$

12. a. $-\frac{12}{13}$ b. $\sqrt{\frac{1}{2}+\frac{1}{\sqrt{13}}}$ c. $-\frac{5}{13}$ d. $\sqrt{\frac{1}{2}-\frac{1}{\sqrt{13}}}$

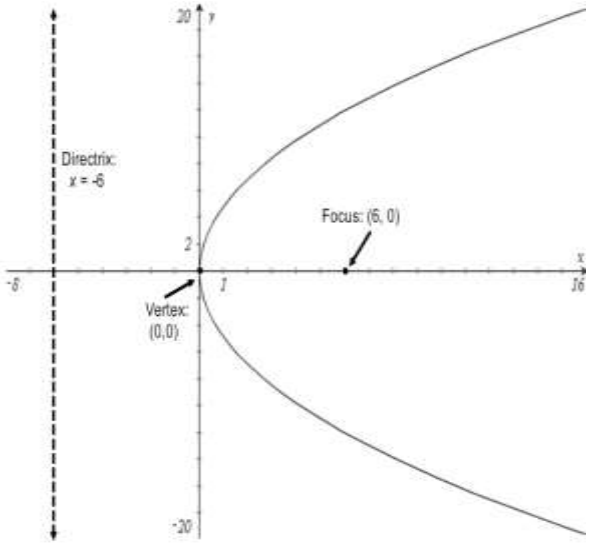
13. a. $z_1 = \left(1,\frac{3\pi}{4}\right)$ or $(1,135°)$; $z_2 = \left(3,\frac{4\pi}{3}\right)$ or $\left(3,240^\circ\right)$ b. $3\left(\cos\frac{25\pi}{12}+i\sin\frac{25\pi}{12}\right)$

c. $3\left(\cos\frac{7\pi}{12}+i\sin\frac{7\pi}{12}\right)$ d. $\frac{1}{3}\left(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right)$

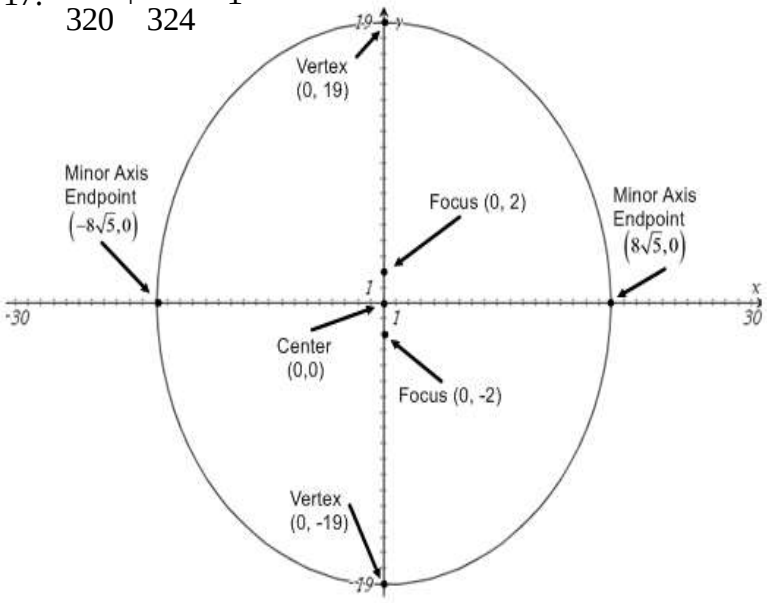
14. a. $8-8i$ b. $-\frac{2187\sqrt{3}}{2}-\frac{2187}{2}i$

15. a. $5\mathbf{i}-\mathbf{j}$ b. $-6\mathbf{i}+11\mathbf{j}$ c. $\sqrt{26}$ d. $\sqrt{157}$ e. 4 f. 60.255°

16. $y^2=24x$



17. $\frac{x^2}{320}+\frac{y^2}{324}=1$



18. Transverse axis: the *x*-axis; Vertices: $(\pm 2,0)$; Foci: $\left(\pm\sqrt{13},0\right)$; Asymptotes: $y=\pm\frac{3}{2}x$