MSLC – Math 1150
Exam 2 Review

Disclaimer: This should NOT be used as your only guide for what to study.

1. Solve the inequalities algebraically.
   a. $(x-1)(x-4)<-2$
   b. $x^3 \geq 2x^2 - x$
   c. $\dfrac{2x-3}{x+1} > 1$
   d. $\dfrac{2x+5}{x+1} > \dfrac{x+1}{x-1}$

2. If $5000 is invested at an interest rate of 7.5% per year, find the amount of the investment at the end of 3 years for the following compounding methods:
   a. Annually
   b. Semiannually
   c. Monthly
   d. Daily
   e. Continuously

3. How much needs to be invested now in order to have $1000 in 5 years given a 4.2% interest rate that is compounded:
   a. daily?
   b. continuously?

4. Find the domain of the following functions:
   a. $f(x) = \log\left(x^2 - 3x - 10\right)$
   b. $g(x) = \dfrac{x}{\log x}$

5. Expand the following logarithms as much as possible.
   a. $\ln\left(2xe^x\right)$
   b. $\log\left[\dfrac{x^2\sqrt{x^2+1}}{(x+3)^4}\right]$
   c. $\ln\left(x\sqrt{x^2-1}\right)$
   d. $\log\left(\dfrac{x^3+2x^2+x}{x^2-3x-4}\right)$

6. Radium-221 has a half-life of 30 seconds. How long will it take for 72% of a sample to decay?

7. Combine the following into a single logarithm.

a. $\ln(a+b)-3\ln c+\ln(a+b)-\ln(a-b)$

b. $2(\log_5 x+2\log_5 y-3\log_5 z)$

8. Express the following using only log or ln.

a. $\log_9\left(x^3\sqrt{1-x}\right)$

b. $\log_\pi\left(\frac{e}{10}\right)$

9. Solve the following equations algebraically.

a. $\log_4(2x-1)=2$

b. $\log_3 x^2+24x=4$

c. $e^{-2x}=4$

d. $\log_6\left(x^2+6x+41\right)=2$

e. $4\cdot 5^{2x+1}=3^{x+3}$

f. $3^{2x}-3^{x+1}=-2$

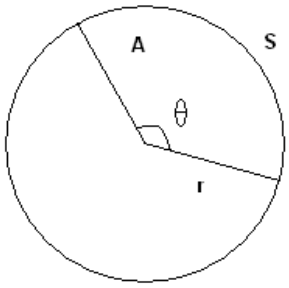
10. The frog population in a small pond grows exponentially. The current population is 70 frogs, and the relative growth rate is 15% per year.

a. Find a function that models the population after t years.

b. Use your equation to find the projected population after 3 years. *Round your answer to the nearest frog.*

c. Use your equation to find the number of years required for the frog population to reach 550 frogs. *Round your answer to two decimal places*.

11. In the circle pictured below, *r* is the radius of the circle, θ is the central angle of the sector, *A* is the area of the sector, and *s* is the length of the arc subtended by the central angle. Find:



a. *A* and *s* if $r=3$ inches, and $\theta=\frac{\pi}{3}$ radians

b. *r* and *s* if $A=2.25$ square miles, and $\theta=36°$

c. *A* and θ if $r=4$ meters, and $s=12.57$ meters

d. *r* and *A* if $s=13$ meters, and $\theta=\frac{\pi}{4}$ radians

12. Write $\cot\theta$ in terms of $\sin\theta$ if θ is in Quadrant IV.

13. Write $\sec\left(\tan^{-1}(x)\right)$ as an algebraic expression in terms of *x*.

14. Find the five remaining trigonometric ratios of θ, given that θ is an acute angle.

a. $a=8, b=15$

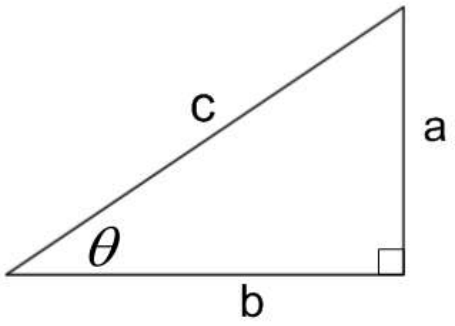
b. $a=12, c=13$

c. $\sin\theta=\frac{2}{7}$

d. $\cot\theta=3$

e. $\cos\theta=\frac{3}{5}$

f. $\csc\theta=\sqrt{5}$



15. Find the five remaining trigonometric ratios of θ, using the information provided.

a. $\csc\theta=-\sqrt{65}$ and $\cot\theta>0$

b. $\sec\theta=-\frac{13}{4\sqrt{3}}$ and $\sin\theta>0$

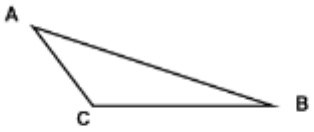
c. $\tan\theta=-\frac{35}{12}$ and $\cos\theta 0$

16. Find the exact value of each of the following.

a. $\tan\left(\sin^{-1}\left(-\frac{99}{101}\right)\right)$

b. $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right)$

17. Given triangle $\triangle ABC$ with the following properties:
$b=14,\ c=17,\ B=44°$ and C is an obtuse angle
Find the measure of angle C.
Round your answer to 2 decimal places.

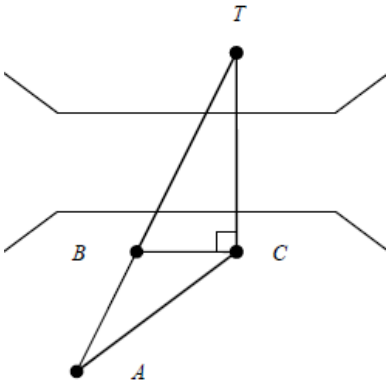


18. A team of surveyors have been hired to measure the distance across a canyon. Using a tree at point T on the opposite site of the canyon as a reference point, they established points A, B, and C and found the following distances:

AB = 12.25 ft
BC = 6.5 ft
AC = 15 ft

a. Find the measure of angle ABC.
Round your answer to 2 decimal places.

b. Find the distance TC across the canyon to the nearest foot.



ANSWERS

1. a. $(2,3)$ b. $[0,\infty)$ c. $(-\infty,-1)\cup(4,\infty)$ d. $(-\infty,-3)\cup(-1,1)\cup(2,\infty)$

2. a. \$6211.48 b. \$6235.89 c. \$6257.23 d. \$6261.47 e. \$6261.61

3. a. \$810.59 b. \$810.58

4. a. Domain: $(-\infty,-2)\cup(5,\infty)$ b. Domain: $(0,1)\cup(1,\infty)$

5. a. $\ln 2+\ln x+x$ b. $2\log x+\frac{1}{2}\log(x^2+1)-4\log(x+3)$

   c. $\ln x+\frac{1}{2}\ln(x+1)+\frac{1}{2}\ln(x-1)$ d. $\log x+\log(x+1)-\log(x-4)$

6. $t\approx 55$ seconds

7. a. $\ln\left[\frac{(a+b)^2}{c^3(a-b)}\right]$ b. $\log_5\left(\frac{x^2y^4}{z^6}\right)$

8. a. $\frac{3\log x}{\log 9}+\frac{\log(1-x)}{2\log 9}$ or $\frac{3\ln x}{\ln 9}+\frac{\ln(1-x)}{2\ln 9}$ b. $\frac{1}{\ln\pi}-\frac{1}{\log\pi}$ (*other correct answers exist*)

9. a. $x=\frac{17}{2}$ b. $x=3$ c. $x=\frac{-\ln 4}{2}$ d. $x=-5$ or $x=-1$

   e. $x=\frac{\ln 4+\ln 5-3\ln 3}{\ln 3-2\ln 5}$ f. $x=0$ or $x=\frac{\ln 2}{\ln 3}$

10. a. $n(t)=70e^{0.15t}$ b. $n(3)\approx 110$ c. $t\approx 13.74$ years

11. a. $s=3.142$ inches $A=4.712$ in^2 b. $r=2.676$ miles $s=1.681$ miles
    c. $\theta=\pi$ (radians) $A=25.13$ m^2 d. $r=16.552$ meters $A=107.589$ m^2

12. $\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$

13. $\sec\theta=\sqrt{1+x^2}$

14. a. $\sin\theta=\frac{8}{17}$ $\cos\theta=\frac{15}{17}$ $\tan\theta=\frac{8}{15}$ $\csc\theta=\frac{17}{8}$ $\sec\theta=\frac{17}{15}$ $\cot\theta=\frac{15}{8}$

b. $\sin\theta=\frac{12}{13}$ $\cos\theta=\frac{5}{13}$ $\tan\theta=\frac{12}{5}$ $\csc\theta=\frac{13}{12}$ $\sec\theta=\frac{13}{5}$ $\cot\theta=\frac{5}{12}$

c. $\sin\theta=\frac{2}{7}$ $\cos\theta=\frac{\sqrt{45}}{7}$ $\tan\theta=\frac{2}{\sqrt{45}}$ $\csc\theta=\frac{7}{2}$ $\sec\theta=\frac{7}{\sqrt{45}}$ $\cot\theta=\frac{\sqrt{45}}{2}$

d. $\sin\theta=\frac{1}{\sqrt{10}}$ $\cos\theta=\frac{3}{\sqrt{10}}$ $\tan\theta=\frac{1}{3}$ $\csc\theta=\sqrt{10}$ $\sec\theta=\frac{\sqrt{10}}{3}$ $\cot\theta=3$

e. $\sin\theta=\frac{4}{5}$ $\cos\theta=\frac{3}{5}$ $\tan\theta=\frac{4}{3}$ $\csc\theta=\frac{5}{4}$ $\sec\theta=\frac{5}{3}$ $\cot\theta=\frac{3}{4}$

f. $\sin\theta=\frac{1}{\sqrt{5}}$ $\cos\theta=\frac{2}{\sqrt{5}}$ $\tan\theta=\frac{1}{2}$ $\csc\theta=\sqrt{5}$ $\sec\theta=\frac{\sqrt{5}}{2}$ $\cot\theta=2$

15. a. θ is in Quadrant III $\sin\theta=-\frac{1}{\sqrt{65}}$; $\cos\theta=-\frac{8}{\sqrt{65}}$; $\tan\theta=\frac{1}{8}$

$\csc\theta=-\sqrt{65}$ $\sec\theta=-\frac{\sqrt{65}}{8}$ $\cot\theta=8$

b. θ is in Quadrant II $\sin\theta=\frac{11}{13}$ $\cos\theta=-\frac{\sqrt{48}}{13}$ $\tan\theta=-\frac{11}{\sqrt{48}}$

$\csc\theta=\frac{13}{11}$ $\sec\theta=-\frac{13}{\sqrt{48}}$ $\cot\theta=-\frac{\sqrt{48}}{11}$

c. θ is in Quadrant IV $\sin\theta=-\frac{35}{37}$ $\cos\theta=\frac{12}{37}$ $\tan\theta=-\frac{35}{12}$

$\csc\theta=-\frac{37}{35}$ $\sec\theta=\frac{37}{12}$ $\cot\theta=-\frac{12}{35}$

16. a. $-\frac{99}{20}$ b. $\frac{2\pi}{3}$

17. $C=122.49°$

18. a. $\angle ABC=101.84°$ b. $TC=31$ feet