MSLC – Math 1150
Exam 3 Review

Disclaimer: This should NOT be used as your only guide for what to study.

1. Establish the identity:
   a. $\dfrac{1-\sin x}{1+\sin x} = (\sec x - \tan x)^2$
   b. $\dfrac{\sec\theta + \csc\theta}{\tan\theta + \cot\theta} = \sin\theta + \cos\theta$
   c. $\dfrac{\tan v \sin v}{\tan v + \sin v} = \dfrac{1-\cos v}{\sin v}$
   d. $\cot(x+y) = \dfrac{\cot x \cot y - 1}{\cot x + \cot y}$
   e. $\sin\left(\dfrac{\pi}{2} - x\right) = \sin\left(\dfrac{\pi}{2} + x\right)$
   f. $\dfrac{1+\sin 2x}{\sin 2x} = 1 + \dfrac{1}{2}\sec x \csc x$

2. Find the exact solution of:
   a. $\sin\dfrac{11\pi}{12}$
   b. $\tan(165°)$
   c. $\cos\dfrac{7\pi}{8}$
   d. $\tan\dfrac{\pi}{12}$

3. Given $\cot x = -\dfrac{2}{3}$ and $\sin x > 0$ find:
   a. $\sin 2x$
   b. $\cos 2x$
   c. $\tan 2x$

4. Find all solutions of:
   a. $\sqrt{2}\cos 2x + 1 = 0$ on the interval $[0, 2\pi)$
   b. $\cos x \sin x - 2\cos x = 0$
   c. $2\sin\dfrac{x}{2} - \sqrt{3} = 0$ on the interval $[0, 8\pi)$

5. Convert to polar coordinates:

a. $-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$

b. $-\frac{3}{2}-\frac{3\sqrt{3}}{2}i$

6. Use DeMoivre's Theorem to find z^7 in the standard $a+bi$ form:

a. $z=1+i$

b. $z=3\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

7. Given $\mathbf{u}=2\mathbf{i}+\mathbf{j}$ and $\mathbf{v}=3\mathbf{i}-2\mathbf{j}$ find:

a. $\mathbf{u}+\mathbf{v}$

b. $3\mathbf{u}-4\mathbf{v}$

c. $|\mathbf{u}+\mathbf{v}|$

d. $|3\mathbf{u}-4\mathbf{v}|$

e. $\mathbf{u}\cdot\mathbf{v}$

f. the angle (in degrees) between **u** and **v.** *Round your answer to 3 decimal places*.

8. Find the complete solution for each of the following systems of linear equations.

a. $\begin{cases}3x+2y=0\\-x-2y=8\end{cases}$

b. $\begin{cases}x-y+2z=0\\3x+y+5z=8\\2x-y-2z=-7\end{cases}$

9. Find the equation of an ellipse with eccentricity of $\frac{1}{9}$ and foci at $(0,\pm 2)$

10. Find the transverse axis, vertices, foci, and the equations of the asymptotes of the hyperbola described by the equation $9x^2-4y^2=36$.

ANSWERS

1a.

$$\frac{1-\sin x}{1+\sin x}=\left(\sec x-\tan x\right)^2$$

$$\frac{1-\sin x}{1+\sin x}\cdot\frac{1-\sin x}{1-\sin x}=RHS$$

$$\frac{1-2\sin x+\sin^2 x}{1-\sin^2 x}=RHS$$

$$\frac{1-2\sin x+\sin^2 x}{\cos^2 x}=RHS$$

$$\frac{1}{\cos^2 x}-\frac{2\sin x}{\cos^2 x}+\frac{\sin^2 x}{\cos^2 x}=RHS$$

$$\sec^2 x-2\sec x\tan x+\tan^2 x=(\sec x-\tan x)^2$$

b.

$$\frac{\sec x+\csc x}{\tan x+\cot x}=\sin x+\cos x$$

$$\frac{\frac{1}{\cos x}+\frac{1}{\sin x}}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}=RHS$$

$$\frac{\frac{\sin x+\cos x}{\cos x\sin x}}{\frac{\sin^2 x+\cos^2 x}{\cos x\sin x}}=RHS$$

$$\frac{\sin x+\cos x}{\cos x\sin x}\cdot\frac{\cos x\sin x}{\sin^2 x+\cos^2 x}=\sin x+\cos x$$

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c.

$$\frac{\tan v\sin v}{\tan v+\sin v}=\frac{1-\cos v}{\sin v}$$

$$\frac{\frac{\sin^2 v}{\cos v}}{\frac{\sin v}{\cos v}+\sin v}=RHS$$

$$\frac{\frac{\sin^2 v}{\cos v}}{\frac{\sin v+\sin v\cos v}{\cos v}}=RHS$$

$$\frac{\sin^2 v}{\sin v+\sin v\cos v}=RHS$$

$$\frac{\sin v}{1+\cos v}=RHS$$

$$\frac{\sin v}{1+\cos v}\cdot\frac{1-\cos v}{1-\cos v}=RHS$$

$$\frac{\sin v(1-\cos v)}{1-\cos^2 v}=RHS$$

$$\frac{\sin v(1-\cos v)}{\sin^2 v}=\frac{1-\cos v}{\sin v}$$

d.

$$\cot(x+y)=\frac{\cot x\cot y-1}{\cot x+\cot y}$$

$$\frac{\cos(x+y)}{\sin(x+y)}=RHS$$

$$\frac{\left(\cos x\cos y-\sin x\sin y\right)}{\left(\sin x\cos y+\cos x\sin y\right)}=RHS$$

$$\frac{\left(\cos x\cos y-\sin x\sin y\right)\frac{1}{\sin y\sin x}}{\left(\sin x\cos y+\cos x\sin y\right)\frac{1}{\sin y\sin x}}=RHS$$

$$\frac{\frac{\cos x\cos y}{\sin x\sin y}-1}{\frac{\cos y}{\sin y}+\frac{\cos x}{\sin x}}=\frac{\cot x\cot y-1}{\cot x+\cot y}$$

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e.

$$\sin\left(\frac{\pi}{2}-x\right)=\sin\left(\frac{\pi}{2}+x\right)$$

$$LHS=\sin\left(\frac{\pi}{2}\right)\cos(x)+\sin x\cos\left(\frac{\pi}{2}\right)$$

$$LHS=1\cdot\cos x+0$$

$$LHS=\cos x$$

$$\sin\left(\frac{\pi}{2}-x\right)=\cos x$$

f.

$$\frac{1+\sin 2x}{\sin 2x}=1+\frac{1}{2}\sec x\csc x$$

$$\frac{\sin 2x}{\sin 2x}+\frac{1}{\sin 2x}=RHS$$

$$1+\frac{1}{2\sin x\cos x}=RHS$$

$$1+\frac{1}{2}\csc x\sec x=1+\frac{1}{2}\sec x\csc x$$

2.a. $\frac{\sqrt{6}-\sqrt{2}}{4}$ b. $-\frac{3-\sqrt{3}}{3+\sqrt{3}}$ c. $-\sqrt{\frac{2+\sqrt{2}}{4}}$ d. $2-\sqrt{3}$

3a. $-\frac{12}{13}$ b. $-\frac{5}{13}$ c. $-\frac{12}{5}$

4. a. $\frac{3\pi}{8}, \frac{5\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}$ b. $\frac{\pi}{2}+\pi k$ c. $\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}$

5. a. $\left(1, \frac{3\pi}{4}\right)$ or $(1, 135°)$ b. $\left(3, \frac{4\pi}{3}\right)$ or $(3, 240°)$ 6. a. $8-8i$ b. $-\frac{2187\sqrt{3}}{2}-\frac{2187}{2}i$

7. a. $5\mathbf{i}-\mathbf{j}$ b. $-6\mathbf{i}+11\mathbf{j}$ c. $\sqrt{26}$ d. $\sqrt{157}$ e. 4 f. 60.255°

8. a. $x=4,\ y=-6$ b. $x=-\frac{17}{23},\ y=\frac{55}{23},\ z=\frac{36}{23}$

9. $\frac{x^2}{320}+\frac{y^2}{324}=1$

10. Transverse axis: the x-axis; Vertices: $(\pm 2, 0)$; Foci: $\left(\pm\sqrt{13}, 0\right)$; Asymptotes: $y=\pm\frac{3}{2}x$