MSLC – Math 1150
Final Exam Review

Disclaimer: This should NOT be used as your only guide for what to study.

1. Use the piece-wise defined function $f(x)=\begin{cases} x^2+2 & \text{if } x\le 0 \\ -2x+4 & \text{if } x\ge 3 \end{cases}$ to answer the following:

   a. Compute *f*(0), *f*(3), *f*(-1), and *f*(4).
   b. Plot the points you found above and sketch a complete graph of $y = f(x)$.

2. Use your graphing calculator to graph each of the functions below over the interval $(-2,2)$ and approximate any local extrema. Also, determine the intervals where the functions are increasing and decreasing.

   Round your answers to three decimal places where appropriate.

   a. $f(x)=x^2$
   b. $g(x)=(x-1)^2(x+1)^2$
   c. $h(x)=\sqrt{|x|}$
   d. $k(x)=x(x-1)(x+1)$

3. Determine the average rate of change of the functions between the given values of *x*.

   a. $h(x)=3x-17$ from $x=-1$ to $x=2$
   b. $f(x)=\dfrac{2x}{3x+1}$ from $x=1$ to $x=t$
   c. $g(x)=\dfrac{1}{\sqrt{x+1}}$ from $x=0$ to $x=a$

4. Write the equation of the function $F(x)=\sqrt{x}$ transformed in the following ways:

   a. shifted 2 units to the left, and shifted up 3 units
   b. reflected about the *x*-axis, then shifted down 3 units
   c. shifted 1 unit to the right, and vertically stretched by a factor of 3
   d. shifted 1 unit to the left, then reflected about the *y*-axis

5. Given $f(x)=\sqrt[3]{x+5}$ and $g(x)=\dfrac{1}{4x+1}$ find:

   a. $f\circ g$
   b. $g\circ f$
   c. $f\circ f$
   d. $g\circ g$

6. Algebraically find the maxima and minima of the functions given.

   a. $f(x)=3x(x-2)+5$
   b. $g(x)=-\dfrac{x^2}{5}+\dfrac{x}{3}-7$

7. Find the inverse of each function, state the domain and range of both the original function and its inverse.

a. $f(x) = 2x - 1$

b. $g(x) = x^3 + 1$

c. $k(x) = \dfrac{1}{x-3}$

d. $h(x) = \dfrac{x}{5-2x}$

8. Find all real zeros and the multiplicity of those zeros for the given polynomials, then find their *y*-intercepts and sketch the graph.

a. $p(x) = (2x-1)(x+1)(x+3)$

b. $p(x) = \frac{1}{4}(x+1)^3(x-3)$

c. $p(x) = (x+1)^2(x-3)^2$

9. Find all zeros (both real and complex) of the following polynomials.

a. $P(x) = x^5 + 7x^3$

b. $P(x) = x^3 - x - 6$

c. $P(x) = x^5 + x^3 + 8x^2 + 8$

10. Factor the polynomials into linear and irreducible quadratic factors with real coefficients.

a. $P(x) = x^4 - 2x^3 - 2x^2 - 2x - 3$

b. $Q(x) = x^4 - x^2 + 2x + 2$

11. Factor the polynomials into linear factors with complex coefficients.

a. $P(x) = x^4 - 2x^3 - 2x^2 - 2x - 3$

b. $Q(x) = x^4 - x^2 + 2x + 2$

12. Find a polynomial with integer coefficients having the following properties:

a. Degree: 3 Zeros: 0, i

b. Degree: 3 Zeros: –3, $1+i$

13. Find all intercepts and asymptotes, then sketch the graph of each rational function.

a. $R(x)=\dfrac{x-2}{x^2-4x}$

b. $R(x)=\dfrac{x^2+3x}{x^2-x-6}$

c. $R(x)=\dfrac{x^3+4}{2x^2+x-1}$

14. Solve the following inequalities. *Write your answer using interval notation.*

a. $5(1-2x)\le 9(x-3)$

b. $\dfrac{2}{3}-\dfrac{1}{2}x\ge\dfrac{1}{6}+x$

c. $(x+3)^2(x+1)<0$

d. $\dfrac{x}{x+1}>3x$

e. $|4x+1|<17$

f. $|2x-1|\ge 5$

15. Solve the equation.

a. $2^{2w+1}=4^{2w-1}$

b. $27^{x+9}=81^{x}$

c. $2\ln(r)=\ln(15r+34)$

d. $\log_4(2x-1)=2$

e. $\log_3\left(x^2+24x\right)=4$

f. $e^{-2x}=4$

g. $\log_6\left(x^2+6x+41\right)=2$

h. $3^{2x}-3^{x+1}=-2$

i. $4\cdot 5^{2x+1}=3^{x+3}$

16. How much needs to be invested now in order to have $1000 in 5 years given a 4.2% interest rate that is compounded:

a. daily?

b. continuously?

17. Determine (correct to 3 decimal places) how long it will take for $2000 to double if it's invested in an account that gives 6% interest compounded:

a) semiannually?

b) quarterly?

c) monthly?

d) daily?

e) continuously?

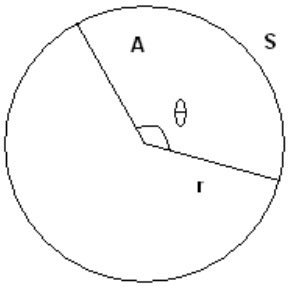
18. The frog population in a small pond grows exponentially. The current population is 70 frogs, and the relative growth rate is 15% per year.

a. Find a function that models the population after t years.

b. Use your equation to find the projected population after 3 years.
Round your answer to the nearest frog.

c. Use your equation to find the number of years required for the frog population to reach 550 frogs. *Round your answer to two decimal places.*

19. In the circle pictured below, r is the radius of the circle, θ is the central angle of the sector, A is the area of the sector, and s is the length of the arc subtended by the central angle. Find:



a. A and s if $r = 3$ inches, and $\theta = \frac{\pi}{3}$ radians

b. r and s if $A = 2.25$ square miles, and $\theta = 36°$

c. A and θ if $r = 4$ meters, and $s = 12.57$ meters

d. r and A if $s = 13$ meters, and $\theta = \frac{\pi}{4}$ radians

20. Use the given information to find the exact value of the six trigonometric functions of the angle θ in the picture.

a) $a = 8,\ b = 15$

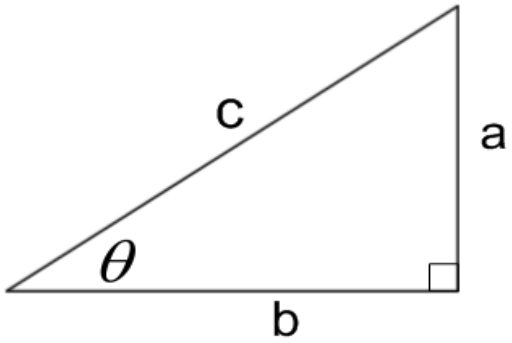
b) $a = 12, c = 13$

c) $\sin\theta = \frac{2}{7}$

d) $\cot\theta = 3$

e) $\cos\theta = \frac{3}{5}$

f) $\csc\theta = \sqrt{5}$



21. For each part, determine the quadrant θ lies in and the values of the five remaining trigonometric functions.

a) $\csc\theta = -\sqrt{65}$ and $\cot\theta > 0$

b) $\sec\theta = -\frac{13}{4\sqrt{3}}$ and $\sin\theta > 0$

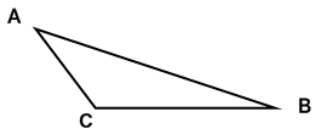
c) $\tan\theta = -\frac{35}{12}$ and $\cos\theta > 0$

22. Find the exact value of each of the following.

a. $\tan\left(\sin^{-1}\left(-\frac{99}{101}\right)\right)$

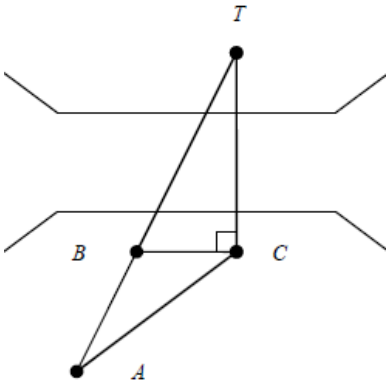
b. $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right)$

23. Given triangle $\triangle ABC$ with the following properties:
$b = 14,\ c = 17,\ B = 44°$ and C is an obtuse angle
Find the measure of angle C.
Round your answer to 2 decimal places.



24. A team of surveyors have been hired to measure the distance across a canyon. Using a tree at point T on the opposite site of the canyon as a reference point, they established points A, B, and C and found the following distances:

AB = 12.25 ft
BC = 6.5 ft
AC = 15 ft



a. Find the measure of angle ABC.
Round your answer to 2 decimal places.

b. Find the distance TC across the canyon to the nearest foot.

25. Establish the identity:

a. $\dfrac{1-\sin x}{1+\sin x} = (\sec x - \tan x)^2$

b. $\dfrac{\sec\theta + \csc\theta}{\tan\theta + \cot\theta} = \sin\theta + \cos\theta$

c. $\dfrac{\tan v \sin v}{\tan v + \sin v} = \dfrac{1-\cos v}{\sin v}$

d. $\cot(x+y) = \dfrac{\cot x \cot y - 1}{\cot x + \cot y}$

e. $\sin\left(\dfrac{\pi}{2} - x\right) = \sin\left(\dfrac{\pi}{2} + x\right)$

f. $\dfrac{1+\sin 2x}{\sin 2x} = 1 + \dfrac{1}{2}\sec x \csc x$

26. Find the exact solution of:

a. $\sin\dfrac{11\pi}{12}$

b. $\tan(165°)$

c. $\cos\dfrac{7\pi}{8}$

d. $\tan\dfrac{\pi}{12}$

27. Given $\cot x = \dfrac{2}{3}$ and $\sin x > 0$ find:

a. $\sin 2x$

b. $\cos 2x$

c. $\tan 2x$

28. Find all solutions of:

a. $\sqrt{2}\cos 2x+1=0$ on the interval $[0,2\pi)$

b. $\cos x\sin x-2\cos x=0$

c. $2\sin\frac{x}{2}-\sqrt{3}=0$ on the interval $[0,8\pi)$

29. Convert to polar coordinates:

a. $-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$

b. $-\frac{3}{2}-\frac{3\sqrt{3}}{2}i$

30. Use DeMoivre's Theorem to find z^7 in the standard $a+bi$ form:

a. $z=1+i$

b. $z=3\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

31. Given $\mathbf{u}=2\mathbf{i}+\mathbf{j}$ and $\mathbf{v}=3\mathbf{i}-2\mathbf{j}$ find:

a. $\mathbf{u}+\mathbf{v}$

b. $3\mathbf{u}-4\mathbf{v}$

c. $|\mathbf{u}+\mathbf{v}|$

d. $|3\mathbf{u}-4\mathbf{v}|$

e. $\mathbf{u}\cdot\mathbf{v}$

f. the angle (in degrees) between **u** and **v.** *Round your answer to 3 decimal places*.

32. Find the complete solution for each of the following systems of linear equations.

a. $\begin{cases}3x+2y=0\\-x-2y=8\end{cases}$

b. $\begin{cases}x-y+2z=0\\3x+y+5z=8\\2x-y-2z=-7\end{cases}$

33. A boat on a river travels downstream for 20 miles in one hour. The return trip back upstream takes 2.5 hours. How fast does the boat travel in still water, and how fast is the current?

34. A fruit stand sells a box of Strawberries for $7 and a box of Kiwi fruit for $10. If they sold a total of 135 boxes of fruit and had revenue of $1110, how many boxes of each fruit did they sell?

35. From the top of a 250 foot tall lighthouse, the angle of depression to a ship on the water is 27°. How far is the ship from the lighthouse? *Round your answer to two decimal places.*

36. Find the equation of the parabola with vertex at the origin and a directrix of $x=-6$.

37. Find the equation of an ellipse with eccentricity of $\frac{1}{9}$ and foci at $(0,\pm 2)$

38. Find the transverse axis, vertices, foci, and the equations of the asymptotes of the hyperbola described by the equation $9x^2-4y^2=36$.

39. Write each of the following sums of a sequence using sigma notation.

a. 2+4+6+…+20

b. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots+\frac{1}{999\cdot 1000}$

40. Find the sum of the following:

a. $\sum_{k=1}^{4} k^2$

b. $\sum_{k=4}^{12} 10$

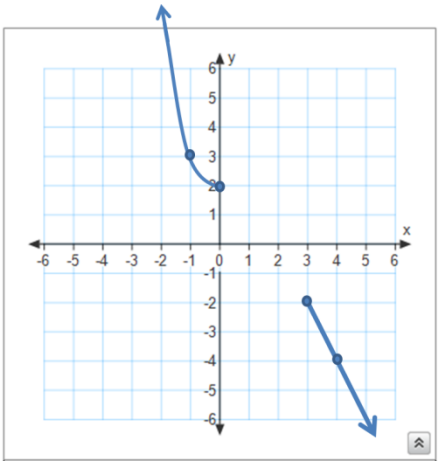
c. $\sum_{n=0}^{20} (1-2n)$

d. $\sum_{j=0}^{5} 7\left(\frac{3}{2}\right)^j$

ANSWERS

1. a) $f(0)=2$; $f(3)=-2$; $f(-1)=3$; $f(4)=-4$

b)



2. a. Min: $(0,0)$; Increasing: $(0,2)$; Decreasing: $(-2,0)$

b. Mins: $(-1,0)$ & $(1,0)$; Max: $(0,1)$; Increasing: $(-1,0)\cup(1,2)$; Decreasing: $(-2,-1)\cup(0,1)$

c. Min: (0,0); Increasing: $(0,2)$; Decreasing: $(-2,0)$

d. Min: $(0.578,-0.385)$; Max: $(-0.578,0.385)$;

Increasing: $(-2,-0.578)\cup(0.578,2)$; Decreasing: $(-0.578,0.578)$

3. a) 3 b) $\frac{1}{6t+2}$ c) $\frac{1-\sqrt{a+1}}{a\sqrt{a+1}}$

4. a) $y=\sqrt{x+2}+3$ b) $y=-\sqrt{x}-3$ c) $y=3\sqrt{x-1}$ d) $y=\sqrt{-(x-1)}$

5.a) $(f\circ g)(x)=\sqrt[3]{\dfrac{20x+6}{4x+1}}$ b) $(g\circ f)(x)=\dfrac{1}{4\sqrt[3]{x+5}+1}$ c) $(f\circ f)(x)=\sqrt[3]{\sqrt[3]{x+5}+5}$ d) $(g\circ g)(x)=\dfrac{4x+1}{4x+5}$

6. a) Min.: $(1,2)$ b) Max.: $\left(\dfrac{5}{6},-\dfrac{247}{36}\right)$

7. a) $f^{-1}(x)=\dfrac{x+1}{2}$ Domain $f(x)=(-\infty,\infty)$; Range $f(x)=(-\infty,\infty)$

Domain $f^{-1}(x)=(-\infty,\infty)$; Range $f^{-1}(x)=(-\infty,\infty)$

b) $g^{-1}(x)=\sqrt[3]{x-1}$ Domain $g(x)=(-\infty,\infty)$; Range $g(x)=(-\infty,\infty)$

Domain $g^{-1}(x)=(-\infty,\infty)$; Range $g^{-1}(x)=(-\infty,\infty)$

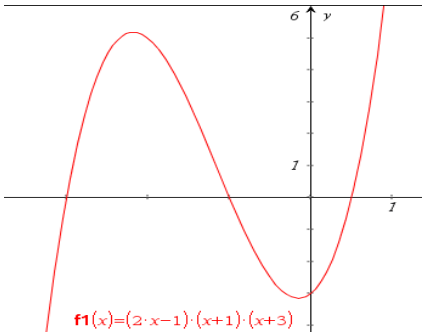
c) $k^{-1}(x)=\dfrac{1}{x}+3$ Domain $k(x)=(-\infty,3)\cup(3,\infty)$; Range $k(x)=(-\infty,0)\cup(0,\infty)$

Domain $k^{-1}(x)=(-\infty,0)\cup(0,\infty)$; Range $k^{-1}(x)=(-\infty,3)\cup(3,\infty)$

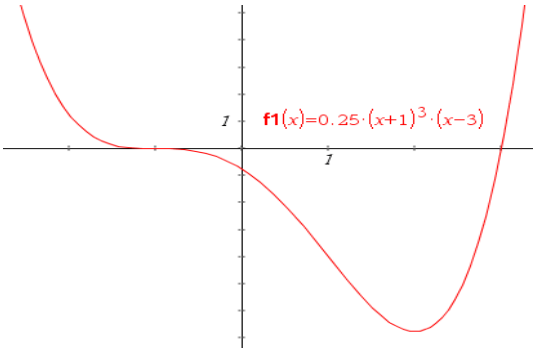
d) $h^{-1}(x)=\dfrac{5x}{5-2x}$ Domain $k(x)=\left(-\infty,\dfrac{5}{2}\right)\cup\left(\dfrac{5}{2},\infty\right)$; Range $k(x)=\left(-\infty,-\dfrac{1}{2}\right)\cup\left(-\dfrac{1}{2},\infty\right)$

Domain $k^{-1}(x)=\left(-\infty,-\dfrac{1}{2}\right)\cup\left(-\dfrac{1}{2},\infty\right)$; Range $k^{-1}(x)=\left(-\infty,\dfrac{5}{2}\right)\cup\left(\dfrac{5}{2},\infty\right)$

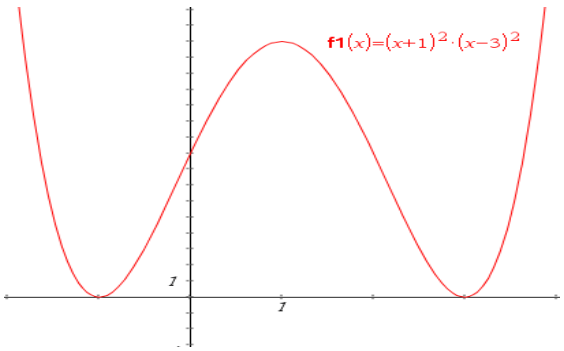
8. a) Zeros: ½ (multiplicity:1);
–1 (multiplicity:1);
–3 (multiplicity:1);
y-int: $(0,-3)$



b) Zeros: –1 (multiplicity:3);
3 (multiplicity:1);
y-int: $(0,-¾)$



c) Zeros: –1 (multiplicity:2);
3 (multiplicity:2);
y-int: $(0,9)$



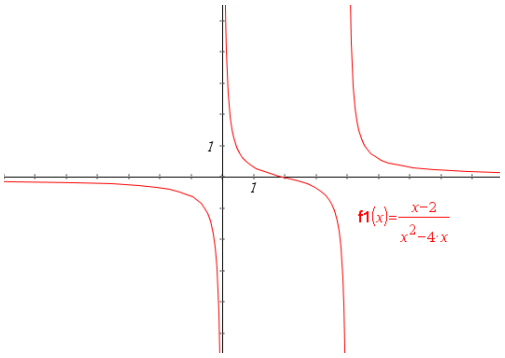
9. a) Zeros: 0; $i\sqrt{7}$; $-i\sqrt{7}$ b) Zeros: 2; $-1+i\sqrt{2}$; $-1-i\sqrt{2}$ c) Zeros: -2; i; $-i$; $1+i\sqrt{3}$; $1-i\sqrt{3}$

10. a) $(x+1)(x-3)(x^2+1)$ b) $(x+1)^2(x^2-2x+2)$

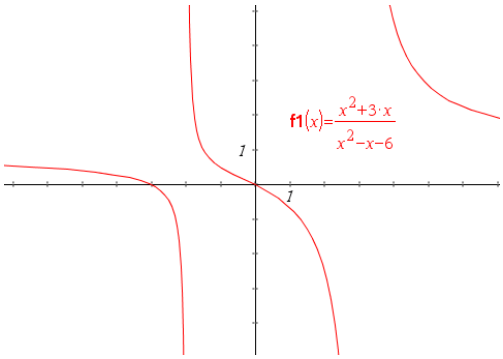
11. a) $(x+1)(x-3)(x+i)(x-i)$ b) $(x+1)^2[x-(1-i)][x-(1+i)]$

12. a) $P(x)=x^3+x$; b) x^3+x^2-4x+6

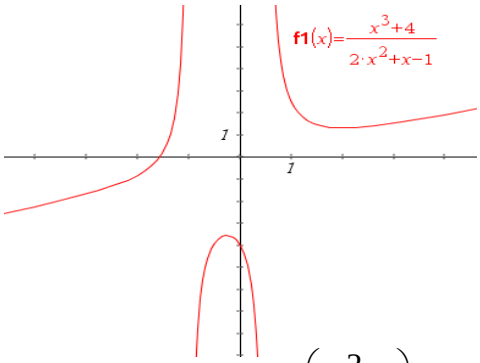
13. a) no y-intercept,
x-intercept: $(2,0)$;
Horizontal Asymptote: $y=0$;
Vertical Asymptotes: $x=0$; $x=4$



b) y-intercept: $(0,0)$
x-intercepts: $(0,0)$ & $(-3,0)$
Horizontal Asymptote: $y=1$
Vertical Asymptotes: $x=-2$; $x=3$



c) y-intercept: $(0,-4)$
x-intercepts: $(\sqrt[3]{-4},0)$
Slant Asymptote: $y=\frac{1}{2}x-\frac{1}{4}$;
Vertical Asymptotes: $x=-1$; $x=\frac{1}{2}$



14. a) $\left[\frac{32}{19},\infty\right)$ b) $\left(-\infty,\frac{1}{3}\right]$ c) $(-\infty,-3)\cup(-3,-1)$ d) $(-\infty,-1)\cup\left(-\frac{2}{3},0\right)$ e) $\left(-\frac{9}{2},4\right)$
f) $(-\infty,-2]\cup[3,\infty)$

15. a. $w=\frac{3}{2}$ b. $x=27$ c. $r=17$ d. $x=\frac{17}{2}$ e. $x=3$ f. $x=\frac{-\ln 4}{2}$
g. $x=-5$ or $x=-1$ h. $x=\frac{\ln 2}{\ln 3}$ or $x=0$ i. $x=\frac{\ln 4+\ln 5-3\ln 3}{\ln 3-2\ln 5}$

16. a. \$810.59 b. \$810.58

17. a) 11.725 yrs b) 11.639 yrs c) 11.581 yrs d) 11.553 yrs e) 11.552 yrs

18. a. $n(t) = 70e^{0.15t}$ b. $n(3) \approx 110$ c. $t \approx 13.74$ years

19. a. $s = 3.142$ inches $A = 4.712$ in^2 b. $r = 2.676$ miles $s = 1.681$ miles
c. $\theta = \pi$ (radians) $A = 25.13$ m^2 d. $r = 16.552$ meters $A = 107.589$ m^2

20. a. $\sin\theta = \frac{8}{17}$ $\cos\theta = \frac{15}{17}$ $\tan\theta = \frac{8}{15}$ $\csc\theta = \frac{17}{8}$ $\sec\theta = \frac{17}{15}$ $\cot\theta = \frac{15}{8}$

b. $\sin\theta = \frac{12}{13}$ $\cos\theta = \frac{5}{13}$ $\tan\theta = \frac{12}{5}$ $\csc\theta = \frac{13}{12}$ $\sec\theta = \frac{13}{5}$ $\cot\theta = \frac{5}{12}$

c. $\sin\theta = \frac{2}{7}$ $\cos\theta = \frac{\sqrt{45}}{7}$ $\tan\theta = \frac{2}{\sqrt{45}}$ $\csc\theta = \frac{7}{2}$ $\sec\theta = \frac{7}{\sqrt{45}}$ $\cot\theta = \frac{\sqrt{45}}{2}$

d. $\sin\theta = \frac{1}{\sqrt{10}}$ $\cos\theta = \frac{3}{\sqrt{10}}$ $\tan\theta = \frac{1}{3}$ $\csc\theta = \sqrt{10}$ $\sec\theta = \frac{\sqrt{10}}{3}$ $\cot\theta = 3$

e. $\sin\theta = \frac{4}{5}$ $\cos\theta = \frac{3}{5}$ $\tan\theta = \frac{4}{3}$ $\csc\theta = \frac{5}{4}$ $\sec\theta = \frac{5}{3}$ $\cot\theta = \frac{3}{4}$

f. $\sin\theta = \frac{1}{\sqrt{5}}$ $\cos\theta = \frac{2}{\sqrt{5}}$ $\tan\theta = \frac{1}{2}$ $\csc\theta = \sqrt{5}$ $\sec\theta = \frac{\sqrt{5}}{2}$ $\cot\theta = 2$

21. a. θ is in Quadrant III; $\sin\theta = -\frac{1}{\sqrt{65}}$ $\cos\theta = -\frac{8}{\sqrt{65}}$ $\tan\theta = \frac{1}{8}$

$\csc\theta = -\sqrt{65}$ $\sec\theta = -\frac{\sqrt{65}}{8}$ $\cot\theta = 8$

b. θ is in Quadrant II; $\sin\theta = \frac{11}{13}$ $\cos\theta = -\frac{\sqrt{48}}{13}$ $\tan\theta = -\frac{11}{\sqrt{48}}$

$\csc\theta = \frac{13}{11}$ $\sec\theta = -\frac{13}{\sqrt{48}}$ $\cot\theta = -\frac{\sqrt{48}}{11}$

c. θ is in Quadrant IV; $\sin\theta = -\frac{35}{37}$ $\cos\theta = \frac{12}{37}$ $\tan\theta = -\frac{35}{12}$

$\csc\theta = -\frac{37}{35}$ $\sec\theta = \frac{37}{12}$ $\cot\theta = -\frac{12}{35}$

22. a. $-\frac{99}{20}$ b. $\frac{2\pi}{3}$

23. $C = 122.49°$

24. a. $\angle ABC = 101.84°$ b. $TC = 31$ feet

25. a.

$$\frac{1-\sin x}{1+\sin x}=\left(\sec x-\tan x\right)^2$$

$$\frac{1-\sin x}{1+\sin x}\cdot\frac{1-\sin x}{1-\sin x}=RHS$$

$$\frac{1-2\sin x+\sin^2 x}{1-\sin^2 x}=RHS$$

$$\frac{1-2\sin x+\sin^2 x}{\cos^2 x}=RHS$$

$$\frac{1}{\cos^2 x}-\frac{2\sin x}{\cos^2 x}+\frac{\sin^2 x}{\cos^2 x}=RHS$$

$$\sec^2 x-2\sec x\tan x+\tan^2 x=(\sec x-\tan x)^2$$

b.

$$\frac{\sec x+\csc x}{\tan x+\cot x}=\sin x+\cos x$$

$$\frac{\frac{1}{\cos x}+\frac{1}{\sin x}}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}=RHS$$

$$\frac{\frac{\sin x+\cos x}{\cos x\sin x}}{\frac{\sin^2 x+\cos^2 x}{\cos x\sin x}}=RHS$$

$$\frac{\sin x+\cos x}{\cos x\sin x}\cdot\frac{\cos x\sin x}{\sin^2 x+\cos^2 x}=\sin x+\cos x$$

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c.

$$\frac{\tan v\sin v}{\tan v+\sin v}=\frac{1-\cos v}{\sin v}$$

$$\frac{\frac{\sin^2 v}{\cos v}}{\frac{\sin v}{\cos v}+\sin v}=RHS$$

$$\frac{\frac{\sin^2 v}{\cos v}}{\frac{\sin v+\sin v\cos v}{\cos v}}=RHS$$

$$\frac{\sin^2 v}{\sin v+\sin v\cos v}=RHS$$

$$\frac{\sin v}{1+\cos v}=RHS$$

$$\frac{\sin v}{1+\cos v}\cdot\frac{1-\cos v}{1-\cos v}=RHS$$

$$\frac{\sin v(1-\cos v)}{1-\cos^2 v}=RHS$$

$$\frac{\sin v(1-\cos v)}{\sin^2 v}=\frac{1-\cos v}{\sin v}$$

d.

$$\cot(x+y)=\frac{\cot x\cot y-1}{\cot x+\cot y}$$

$$\frac{\cos(x+y)}{\sin(x+y)}=RHS$$

$$\frac{\left(\cos x\cos y-\sin x\sin y\right)}{\left(\sin x\cos y+\cos x\sin y\right)}=RHS$$

$$\frac{\left(\cos x\cos y-\sin x\sin y\right)\frac{1}{\sin y\sin x}}{\left(\sin x\cos y+\cos x\sin y\right)\frac{1}{\sin y\sin x}}=RHS$$

$$\frac{\frac{\cos x\cos y}{\sin x\sin y}-1}{\frac{\cos y}{\sin y}+\frac{\cos x}{\sin x}}=\frac{\cot x\cot y-1}{\cot x+\cot y}$$

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e.

$$\sin\left(\frac{\pi}{2}-x\right)=\sin\left(\frac{\pi}{2}+x\right)$$

$$LHS=\sin\left(\frac{\pi}{2}\right)\cos(x)+\sin x\cos\left(\frac{\pi}{2}\right)$$

$$LHS=1\cdot\cos x+0$$

$$LHS=\cos x$$

$$\sin\left(\frac{\pi}{2}-x\right)=\cos x$$

f.

$$\frac{1+\sin 2x}{\sin 2x}=1+\frac{1}{2}\sec x\csc x$$

$$\frac{\sin 2x}{\sin 2x}+\frac{1}{\sin 2x}=RHS$$

$$1+\frac{1}{2\sin x\cos x}=RHS$$

$$1+\frac{1}{2}\csc x\sec x=1+\frac{1}{2}\sec x\csc x$$

26. a. $\frac{\sqrt{6}-\sqrt{2}}{4}$ b. $-\frac{3-\sqrt{3}}{3+\sqrt{3}}$ c. $-\sqrt{\frac{2+\sqrt{2}}{4}}$ d. $2-\sqrt{3}$

27. a. $\frac{12}{13}$ b. $-\frac{5}{13}$ c. $-\frac{12}{5}$

28. a. $\frac{3\pi}{8}, \frac{5\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}$ b. $\frac{\pi}{2}+\pi k$ c. $\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}$

29. a. $\left(1, \frac{3\pi}{4}\right)$ or $(1, 135°)$ b. $\left(3, \frac{4\pi}{3}\right)$ or $(3, 240°)$

30. a. $8-8i$ b. $-\frac{2187\sqrt{3}}{2}-\frac{2187}{2}i$

31. a. $5\mathbf{i}-\mathbf{j}$ b. $-6\mathbf{i}+11\mathbf{j}$ c. $\sqrt{26}$ d. $\sqrt{157}$ e. 4 f. 60.255°

32. a. $x=4,\ y=-6$ b. $x=-\frac{17}{23},\ y=\frac{55}{23},\ z=\frac{36}{23}$

33. Boat's speed in still water = 14mph; Current's speed = 6mph

34. 80 boxes of Strawberries; 55 boxes of Kiwi fruit

35. 490.65 feet

36. $y^2=24x$

37. $\frac{x^2}{320}+\frac{y^2}{324}=1$

38. Transverse axis: the x-axis; Vertices: $(\pm 2, 0)$; Foci: $\left(\pm\sqrt{13}, 0\right)$; Asymptotes: $y=\pm\frac{3}{2}x$

39. a. $\sum_{k=1}^{10} 2k$ b. $\sum_{k=1}^{999} \frac{1}{k(k+1)}$

40. a. 30 b. 90 c. –399 d. $\frac{4655}{32}$