MSLC - Math 1172 Exam 2 Review

1. Find the following integrals:
   a) \( \int \frac{x + 5}{(x-1)(x+2)} \, dx \)
   b) \( \int_1^4 \frac{x^3 + x^2 + x + 1}{x^2 + 4x + 3} \, dx \)
   c) \( \int_1^\infty \frac{1}{x^4 + x^2} \, dx \)
   d) \( \int_{\ln 3}^{\ln 4} \frac{4e^x}{3e^{2x} - 2e^x - 1} \, dx \)
   e) \( \int_1^\infty \frac{e}{x \ln^2 x} \, dx \)
   f) \( \int_{-\infty}^{\infty} e^{-|x|} \, dx \)

2. Considering the following sequences.
   a. Find the limit of the sequence \( \{a_n\} \) if it exists.
   b. Determine if the associated series \( \sum_{n=1}^\infty a_n \) converges or diverges. If the series is geometric and convergent, find the value of the series.
   a) \( \left\{ \frac{5^n}{n} \right\}_{n=1}^\infty \)
   b) \( \left\{ \frac{3^n}{10^{n+5}} \right\}_{n=3}^\infty \)
   c) \( \left\{ \frac{n!}{n+1} \right\}_{n=1}^\infty \)
   d) \( \left\{ \frac{n^2 - 3}{4n(n+1)} \right\}_{n=3}^\infty \)
   e) \( \left\{ \frac{1}{\sqrt{n+1}} \right\}_{n=2}^\infty \)

3. Let \( f(x) = \cos(5x) \)
   a. Find the quadratic approximating polynomial of \( f(x) \) centered at \( x = 0 \).
   b. Use the quadratic from (a) to estimate \( \cos(1) \).
   c. Estimate the maximum absolute error in your approximation.
   d. How many terms are needed in an approximating polynomial to ensure the error is less than \( 10^{-3} \)?
   e. Find the Maclaurin series for \( f(x) \).
   f. Find the radius of convergence for the Maclaurin series in (e).
   g. Use your answer to part (e) to find the Maclaurin series for \( \sin(5x) \)

4. Let \( f(x) = x^4 e^x \)
   a. Find the first four terms of the approximating polynomial of \( f(x) \) centered at \( x = 0 \).
   b. Use the approximating polynomial from (a) to estimate \( 16e^2 \).
   c. Estimate the maximum absolute error in your approximation.
   d. Find the MacLaurin series for \( f(x) = x^4 e^x \)
   e. Find the radius of convergence for the MacLaurin series in (e).
   f. Use your answer to part (e) to find the MacLaurin series for \( 4x^3 e^x + x^4 e^x \)

5. Let \( f(x) = x^2 + 5x + 3 \)
   a. Find the Taylor series for \( f(x) \) centered at \( x = 2 \).
   b. Use the Taylor Series from (a) to estimate \( f(1) \).
c. Find the absolute error in your approximation.

6. Consider the power series \( \sum_{k=1}^{\infty} \frac{x^{k+4}}{(k+3)2^k} \). Find the radius of convergence and find the function represented by the power series.